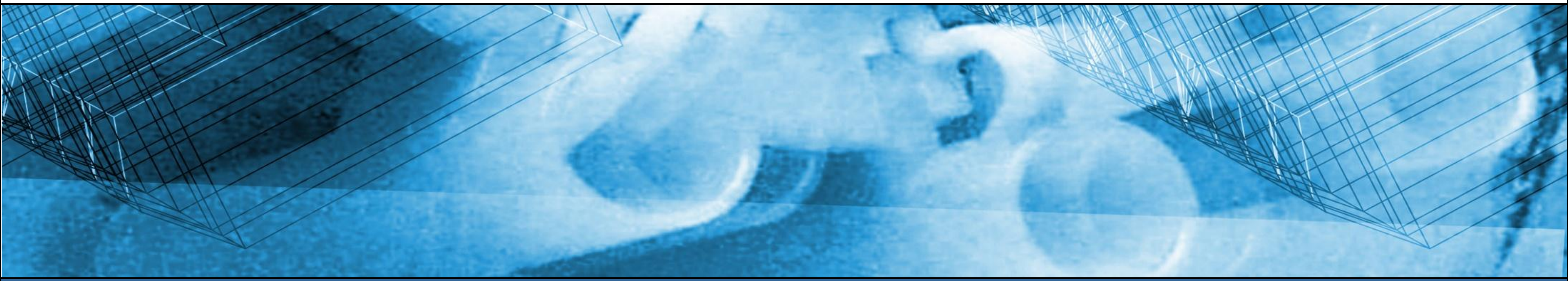
rle AT MIT



Quantitative Susceptibility Mapping with Magnitude Prior

Berkin Bilgic[1], Audrey P. Fan[1], Elfar Adalsteinsson[1,2]

[1]EECS, MIT, Cambridge, MA, United States

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Quantitative Susceptibility Mapping (QSM)

- Quantitative Susceptibility Mapping (QSM) aims to quantify tissue magnetic susceptibility with applications such as,
  - Tissue contrast enhancement[1]
  - Estimation of venous blood oxygenation[2]
  - Quantification of tissue iron concentration[3]

[1] Duyn JH *et al*., PNAS 2007 [2] Fan AP *et al*., ISMRM 2010 [3] Liu T *et al*., ISMRM 2010

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- Estimation of the susceptibility map χ from the unwrapped phase φ involves solving an inverse problem,

$$\delta = \mathbf{F}^{-1}\mathbf{D}\mathbf{F}\chi$$

F: Discrete Fourier Transform matrix

D: susceptibility kernel in k-space

$$\delta = \frac{\varphi}{\gamma \cdot \boldsymbol{TE} \cdot \boldsymbol{B}_0}$$: normalized field map

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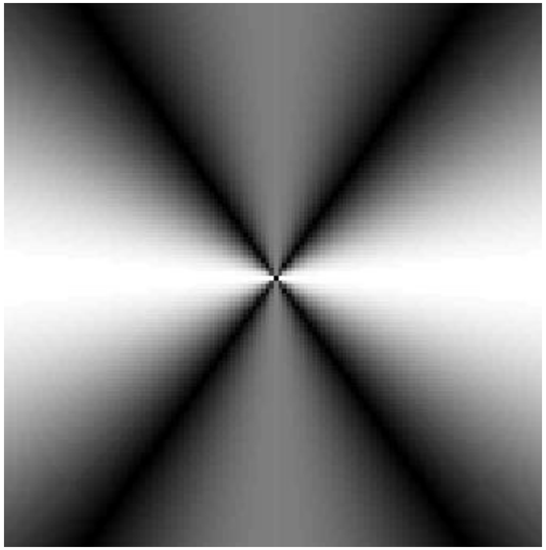
- Estimation of the susceptibility map χ from the unwrapped phase φ involves solving an inverse problem, $\delta = \mathbf{F}^{-1}\mathbf{D}\mathbf{F}\chi$

- The inversion is made difficult by zeros on a conical surface in susceptibility kernel $\mathbf{D}$

$$\mathbf{D} = \left(\frac{1}{3} - \frac{\boldsymbol{k}_z^2}{\boldsymbol{k}^2} \right)$$

$|\mathbf{D}|$



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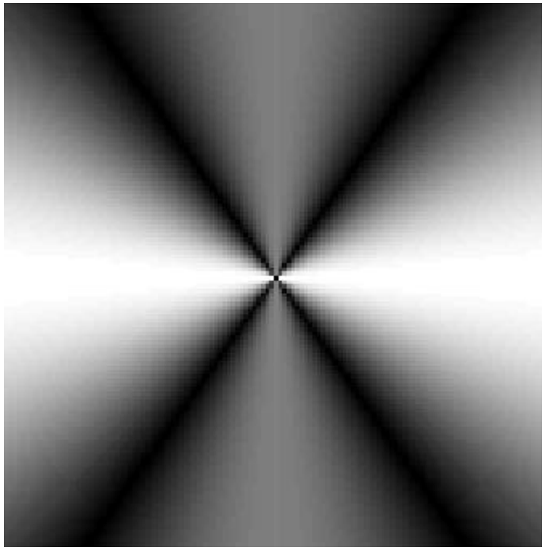
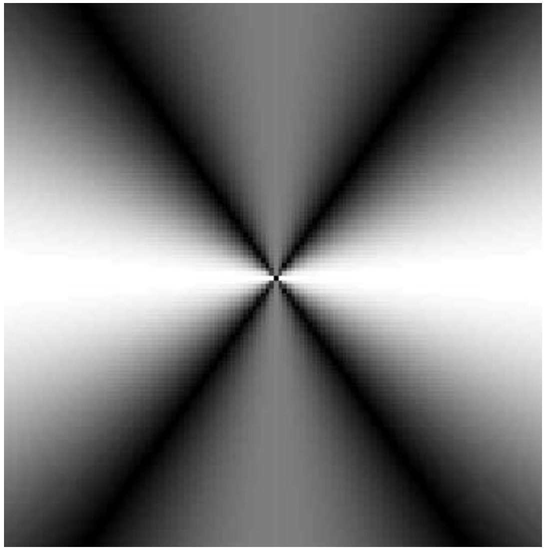
Regularized Inversion for QSM

$|\mathbf{D}|$

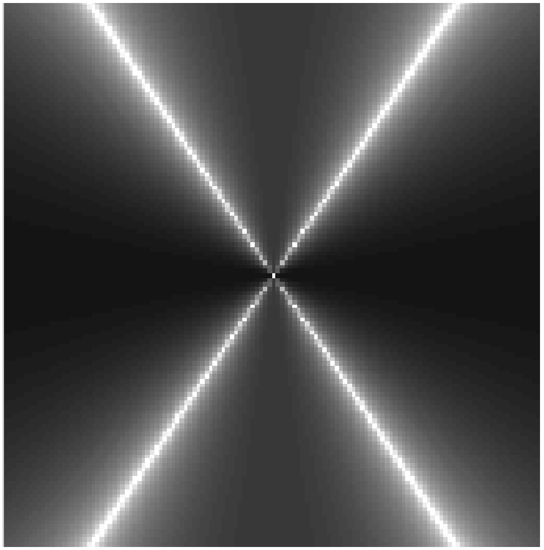


Regularized Inversion for QSM

$|\mathbf{D}|$



$\log|\mathbf{D}^{-1}|$

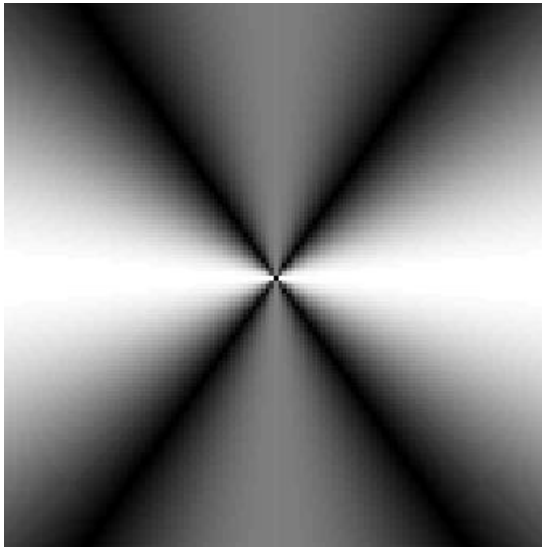


$$\mathbf{F}\mathbf{D}^{-1}\mathbf{F}\delta = \chi$$

- Solving for χ by convolving with the inverse of $\mathbf{D}$ is not possible, as it diverges along the magic angle

Regularized Inversion for QSM

$|\mathbf{D}|$



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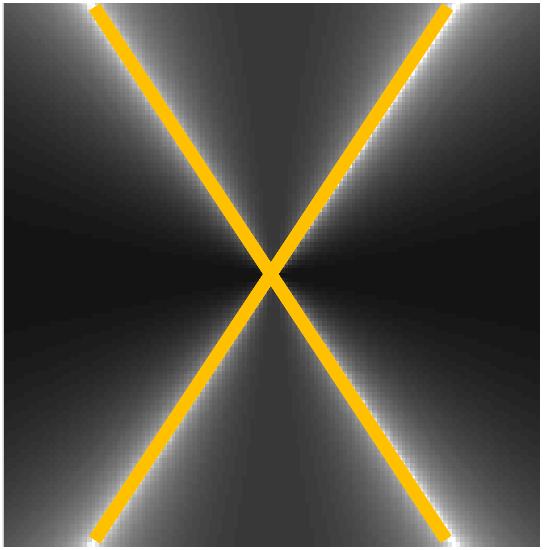
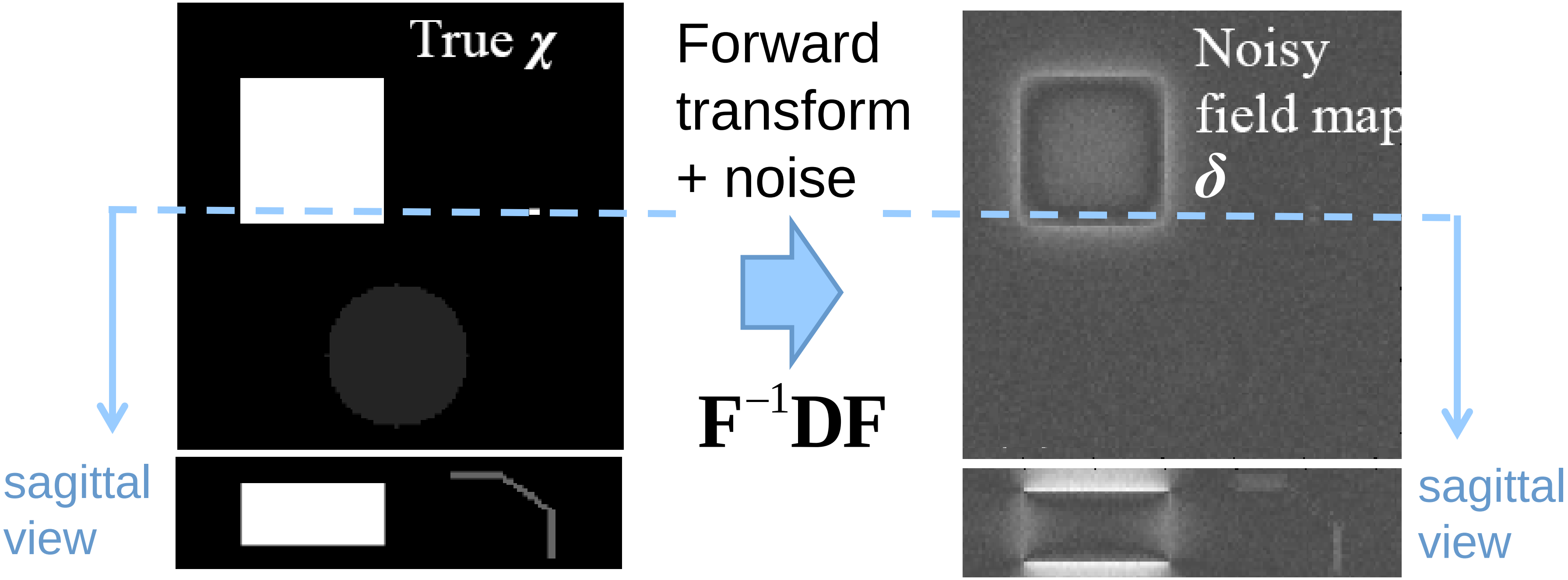


$$\xcancel{\mathbf{F}\mathbf{D}^{-1}\mathbf{F}\,\delta} = \chi$$

diverges to ∞

- Solving for χ by convolving with the inverse of $\mathbf{D}$ is not possible, as it diverges along the magic angle

Regularized Inversion for QSM



- Solving for χ by convolving with the inverse of $\mathbf{D}$ is not possible, as it diverges along the magic angle
- Spatial details that have frequency components at the magic angle lose conspicuity in the field map δ

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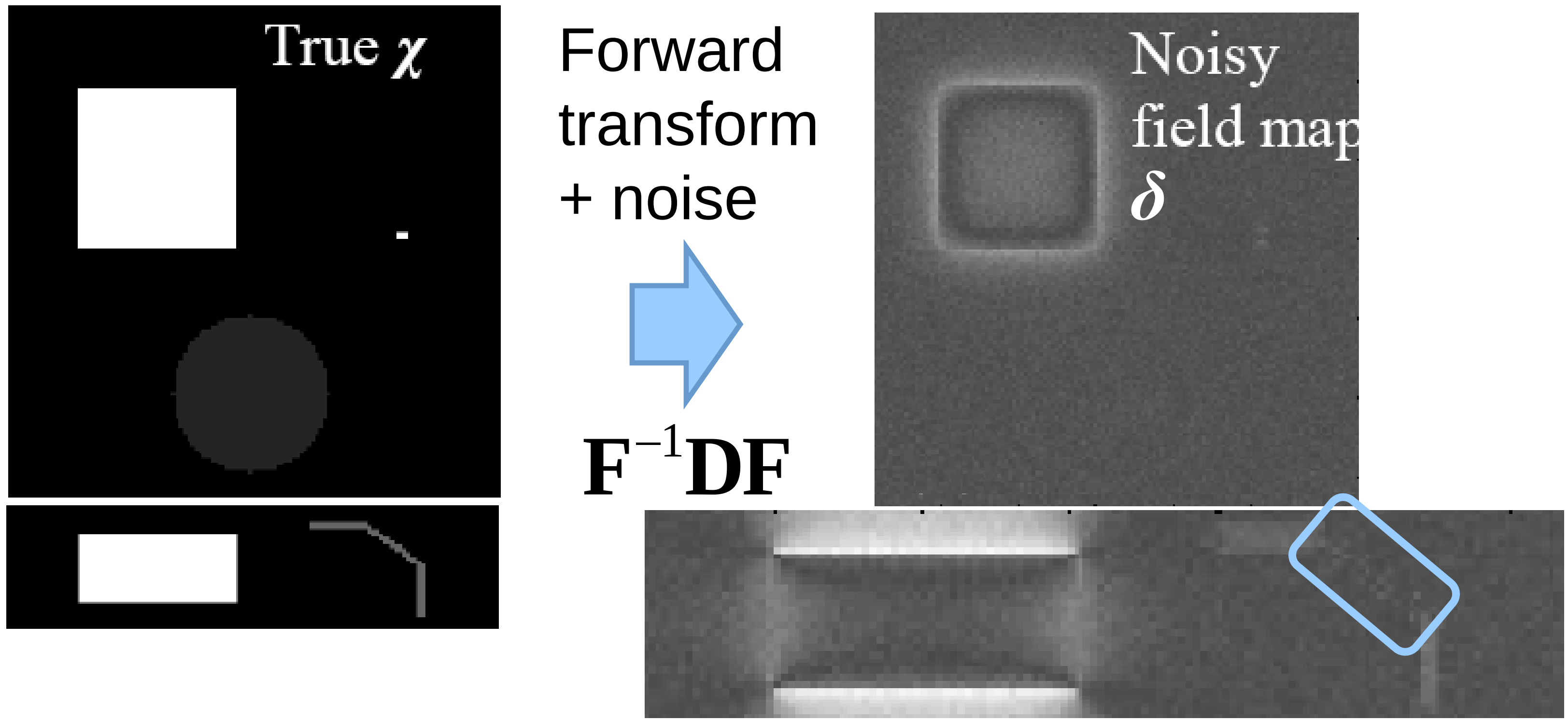


- Solving for χ by convolving with the inverse of $\mathbf{D}$ is not possible, as it diverges along the magic angle
- Spatial details that have frequency components at the magic angle lose conspicuity in the field map δ
- We propose to use regularization to facilitate the inversion

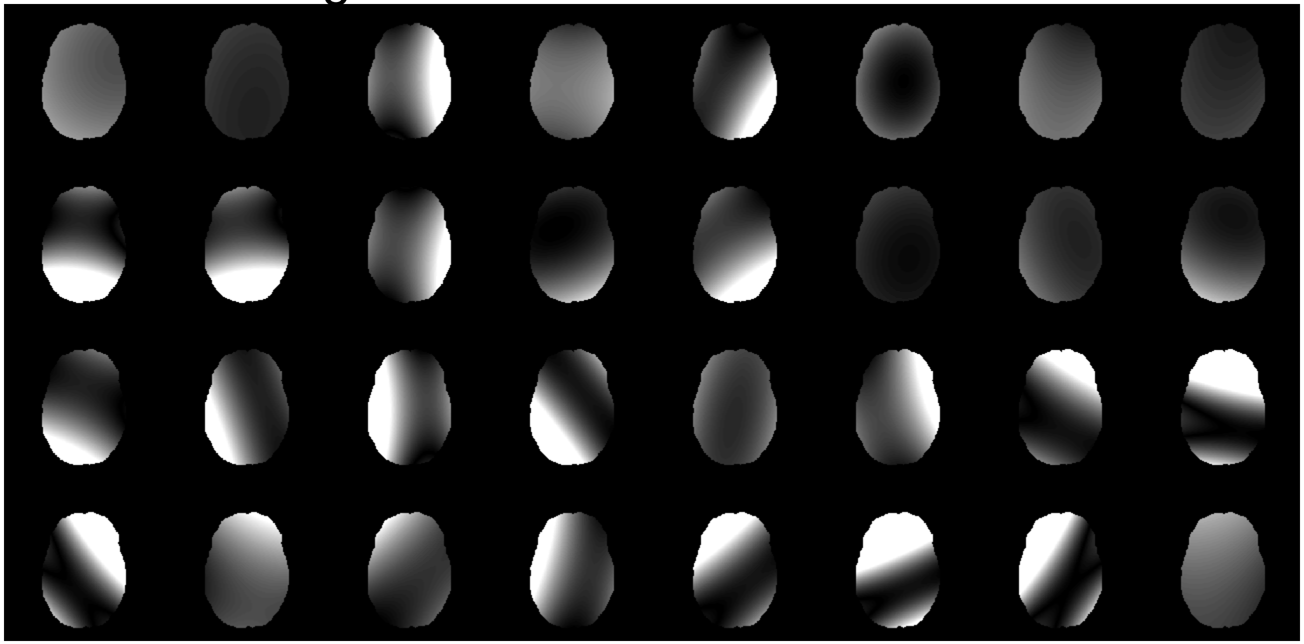
Phase-aware Coil Combination

- 3D GRE acquisition with phased array coils and body coil
- Normalize each channel image with the body coil

Coil Combination → Brain Mask Extraction (BET) → Phase Unwrapping (PRELUDE) → Background Phase Removal → QSM with FOCUSS

Phase-aware Coil Combination

magnitudes of the coil sensitivities



- 3D GRE acquisition with phased array coils and body coil
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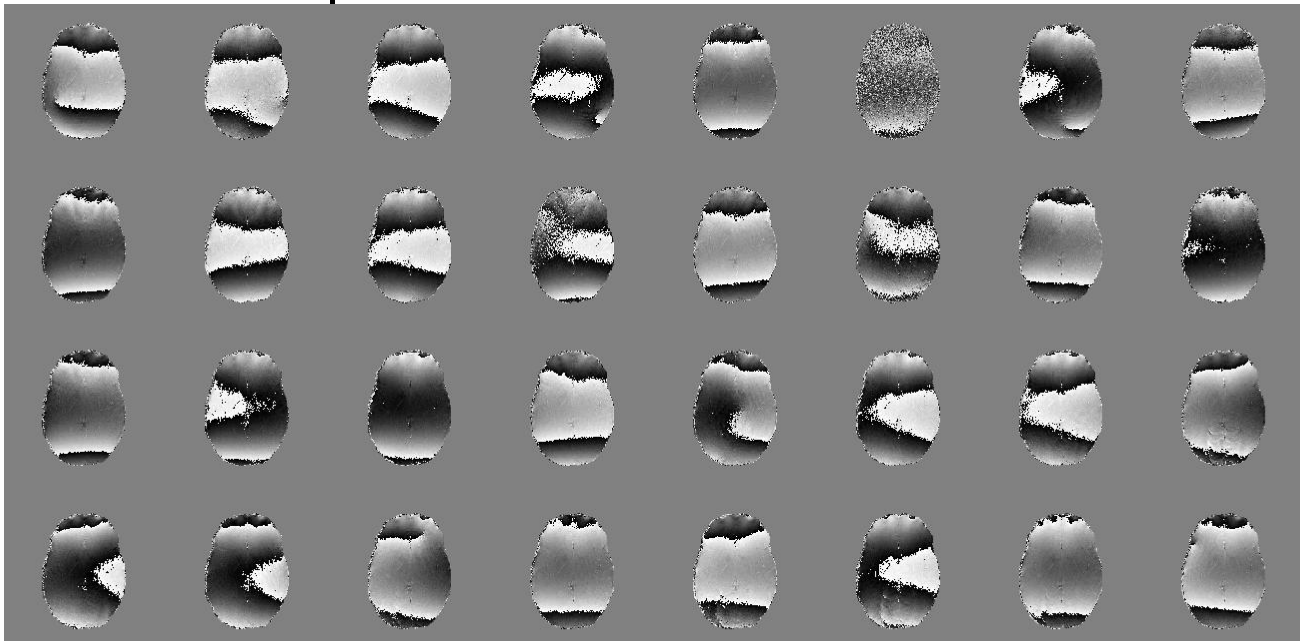
Phase-aware Coil Combination

phase of the coil sensitivities



- 3D GRE acquisition with phased array coils and body coil
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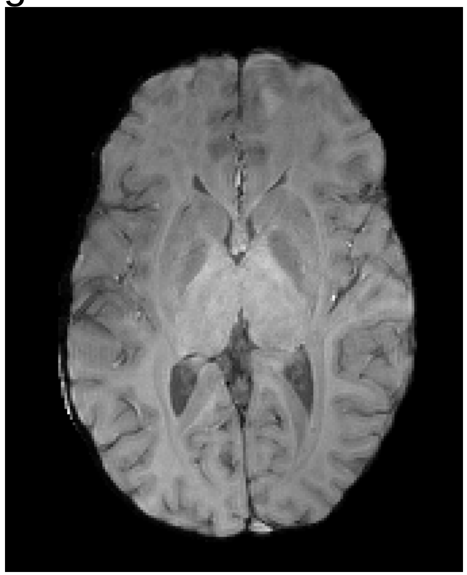
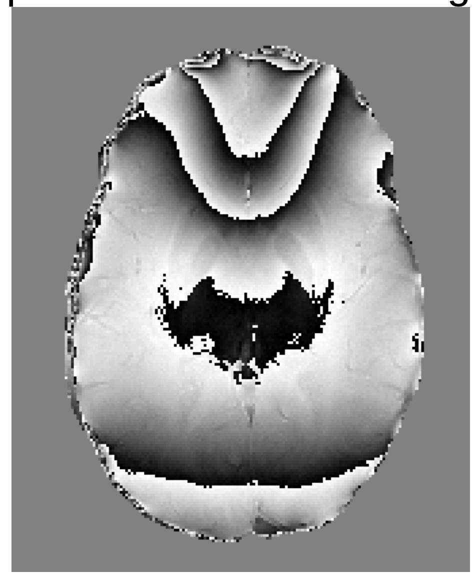
Phase-aware Coil Combination

magnitude of combined image



phase of combined image



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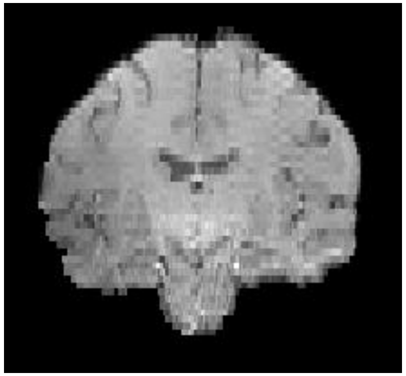
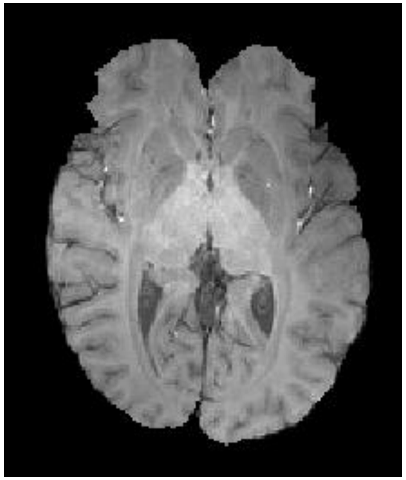
- Phase of the normalized images → phase of the coil sensitivities
- Final image is obtained by least-squares coil combination

Coil Combination → Brain Mask Extraction (BET) → Phase Unwrapping (PRELUDE) → Background Phase Removal → QSM with FOCUSS

Brain Mask Extraction & Phase Unwrapping

- Brain mask was generated with the FSL Brain Extraction Tool[1]





[1] Smith SM, Hum. Brain Mapp. 2002

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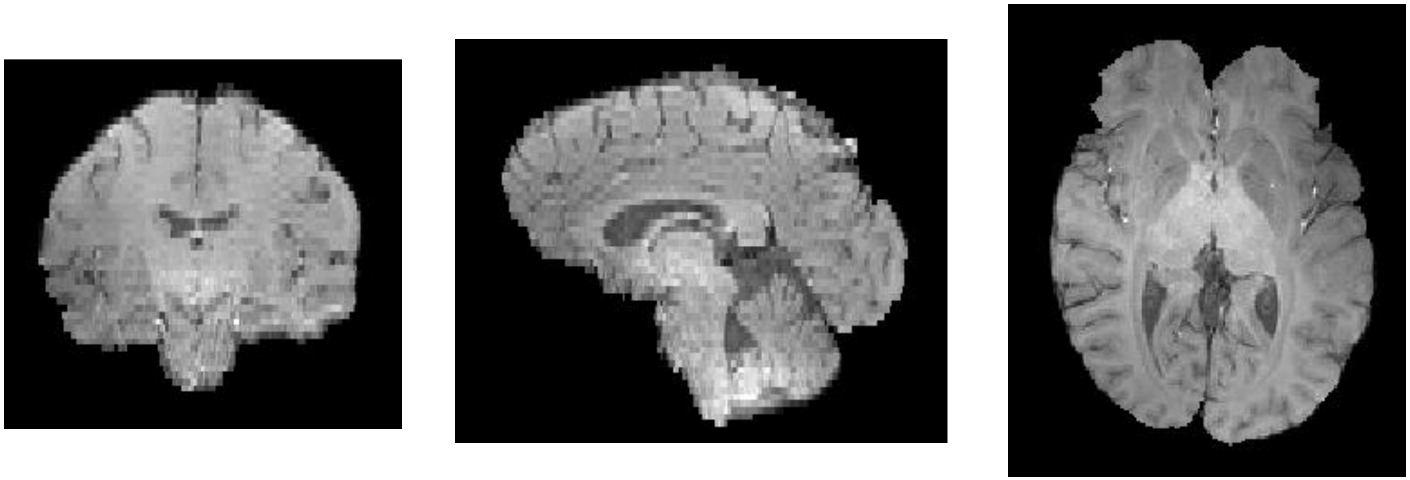
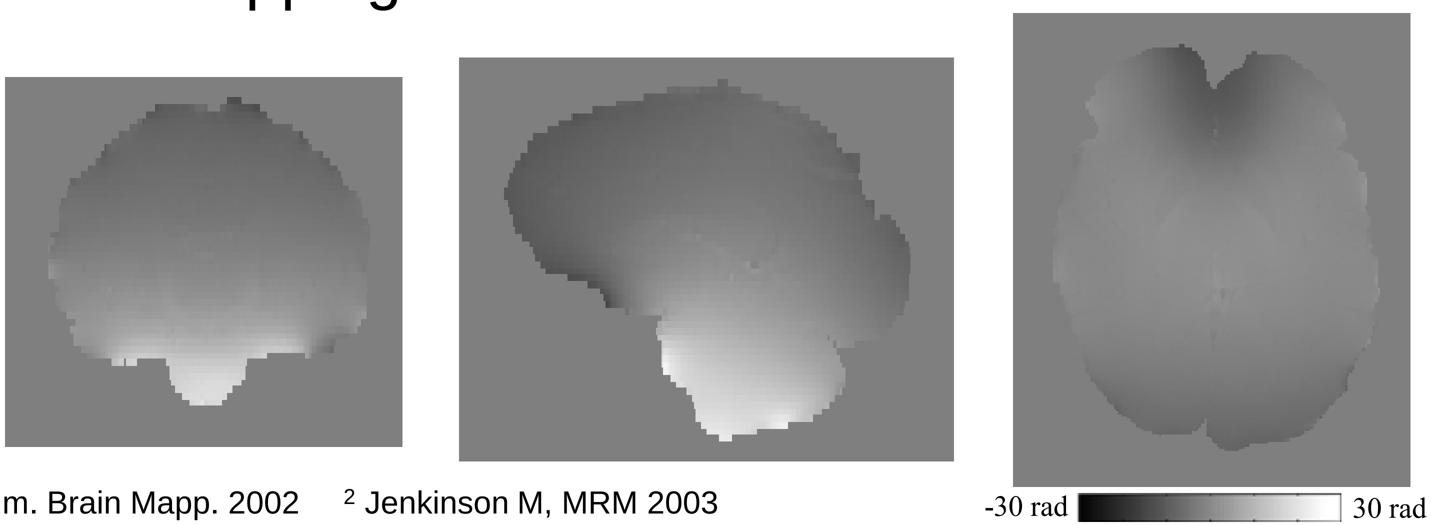


- Phase unwrapping was done with the FSL PRELUDE[2]



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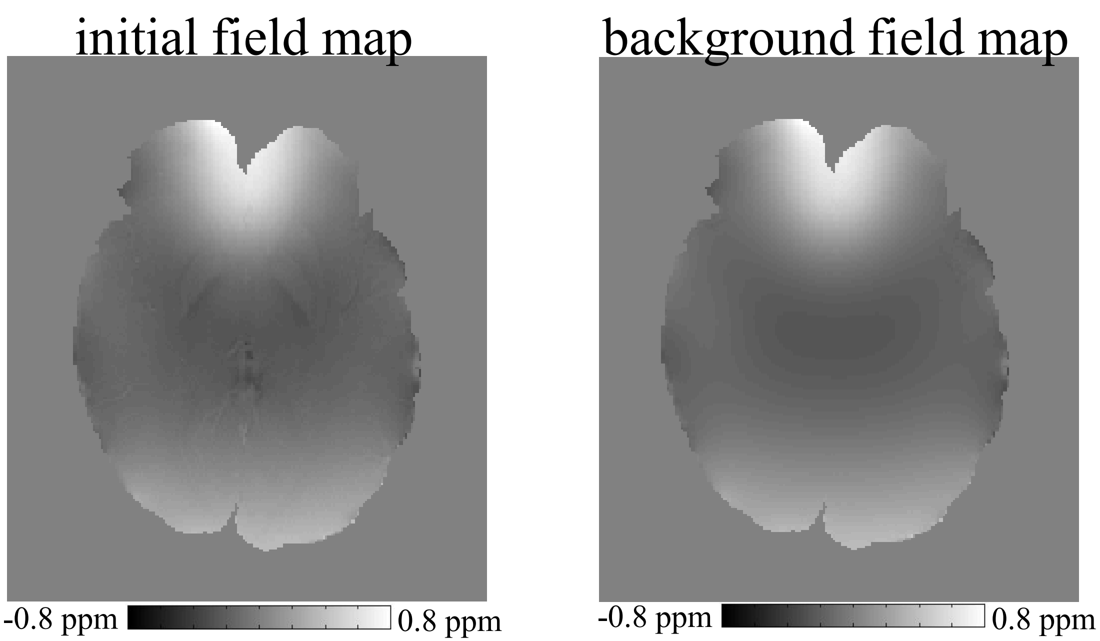
Background Phase Removal

- The background phase was estimated with the Effective Dipole Fitting method[1]

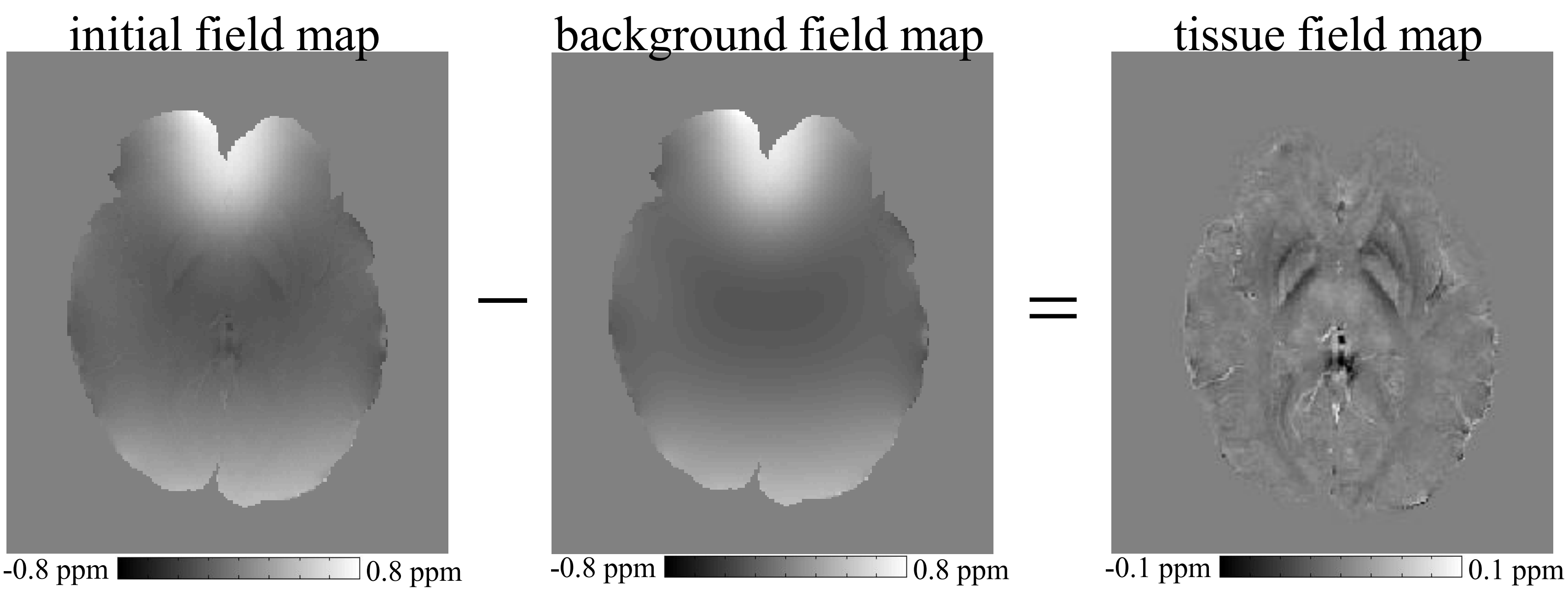


[1] Liu T *et al.*, ISMRM 2010

Background Phase Removal

- The background phase was estimated with the Effective Dipole Fitting method[1]

- Subtracting the estimated background from the initial field map gives the tissue field map



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FOCUSS-QSM with magnitude prior

- The tissue field map δ is related to the susceptibility distribution χ via

$$\delta = \mathbf{F}^{-1}\mathbf{D}\mathbf{F}\chi$$

FOCUSS-QSM with magnitude prior

- The tissue field map $\boldsymbol{\delta}$ is related to the susceptibility distribution $\boldsymbol{\chi}$ via

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- Multiplying both sides with $\mathbf{V}_x\mathbf{F}$

$$\mathbf{V}_{\boldsymbol{x}}\mathbf{F}\delta = \mathbf{V}_{\boldsymbol{x}}\mathbf{D}\mathbf{F}\chi$$

where $\mathbf{V}_{\boldsymbol{x}}$ is a diagonal matrix with $\mathbf{V}_{\boldsymbol{x}}\left(\omega,\omega\right)=\left(1-\boldsymbol{e}^{-2\pi\boldsymbol{j}\omega/\boldsymbol{n}}\right)$

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- This corresponds to taking the spatial gradient along the x axis

$$\mathbf{F}\left(\partial_{\boldsymbol{x}}\delta\right) = \mathbf{D}\mathbf{F}\left(\partial_{\boldsymbol{x}}\chi\right)$$

FOCUSS-QSM with magnitude prior

- The gradient of the tissue field map $\boldsymbol{\delta}$ is related to the gradient of the susceptibility distribution $\boldsymbol{\chi}$ via

$$\mathbf{F}\left(\partial_{\boldsymbol{x}}\delta\right)=\mathbf{DF}\left(\partial_{\boldsymbol{x}}\chi\right)$$

- We solve for $\partial_{\boldsymbol{x}}\chi$ with the FOCUSS algorithm[1]

at k^{th} iteration,

$$\mathbf{W}_{\boldsymbol{k}}=\operatorname{diag}\left(\left|\partial_{\boldsymbol{x}}\chi_{\boldsymbol{k}-1}\right|^{1/2}\right)$$

[1] Gorodnitsky IF *et al.*, IEEE T. Signal Process. 1997

Coil Combination → Brain Mask Extraction (BET) → Phase Unwrapping (PRELUDE) → Background Phase Removal → **QSM with FOCUSS**

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$$\boldsymbol{q}_{\boldsymbol{k}}=\underset{\boldsymbol{q}}{\operatorname{argmin}}\left\|\mathbf{F}\left(\partial_{\boldsymbol{x}}\delta\right)-\mathbf{DFW}_{\boldsymbol{k}}\boldsymbol{q}\right\|_2^2+\lambda\left\|\boldsymbol{q}\right\|_2^2$$

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FOCUSS-QSM with magnitude prior

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- To impose this prior, we modify the update equations as,

$$\mathbf{W}_{prior} = \mathrm{diag}\left(\left|\partial_x \boldsymbol{m}\right|^{1/2}\right), \quad \boldsymbol{m}\text{: magnitude image}$$

at k^{th} iteration,

$$\mathbf{W}_k = \mathrm{diag}\left(\left|\partial_x \chi_{k-1}\right|^{1/2}\right)$$

$$\boldsymbol{q}_k = \underset{\boldsymbol{q}}{\mathrm{argmin}} \left\| \mathbf{F}(\partial_x \delta) - \mathbf{D}\mathbf{F}\mathbf{W}_{prior}\mathbf{W}_k \boldsymbol{q} \right\|_2^2 + \lambda \left\| \boldsymbol{q} \right\|_2^2$$

$$\partial_x \chi_k = \mathbf{W}_{prior}\mathbf{W}_k \boldsymbol{q}_k$$

FOCUSS-QSM with magnitude prior

- We expect the susceptibility distribution to share similar spatial gradients as the magnitude image.

- Expressed in terms of $\partial_x \chi$,

$$\mathbf{W}_{prior} = \mathrm{diag}\left(\left|\partial_x \boldsymbol{m}\right|^{1/2}\right), \quad \boldsymbol{m}\text{: magnitude image}$$

$$\partial_x \chi_k = \underset{\partial_x \chi}{\mathrm{argmin}} \left\| \mathbf{F}\left(\partial_x \delta\right) - \mathbf{DF}\left(\partial_x \chi\right)\right\|_2^2 + \lambda \left\| \mathbf{W}_{prior}^{-1} \mathbf{W}_k^{-1} \left(\partial_x \chi\right)\right\|_2^2$$

FOCUSS-QSM with magnitude prior

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$$\partial_x \chi_k = \underset{\partial_x \chi}{\mathrm{argmin}} \left\|\mathbf{F}(\partial_x \delta) - \mathbf{DF}(\partial_x \chi)\right\|_2^2 + \lambda \boxed{\left\|\mathbf{W}_{prior}^{-1} \mathbf{W}_k^{-1} (\partial_x \chi)\right\|_2^2}$$

if $\partial_x \boldsymbol{m}_i$ is small, $\mathbf{W}_{prior}^{-1}(\boldsymbol{i},\boldsymbol{i})$ will be large and penalize $\partial_x \chi_i$ more

FOCUSS-QSM with magnitude prior

- After estimating the spatial gradients along x, y and z axes, the susceptibility distribution that matches these is found by solving a least squares problem,

$$\chi = \underset{\theta}{\operatorname{argmin}} \sum_{\boldsymbol{r}=\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \left\| \partial_{\boldsymbol{r}} \theta - \partial_{\boldsymbol{r}} \chi \right\|_2^2 + \beta \cdot \left\| \delta - \mathbf{F}^{-1} \mathbf{D} \mathbf{F} \theta \right\|_2^2$$

FOCUSS-QSM with magnitude prior

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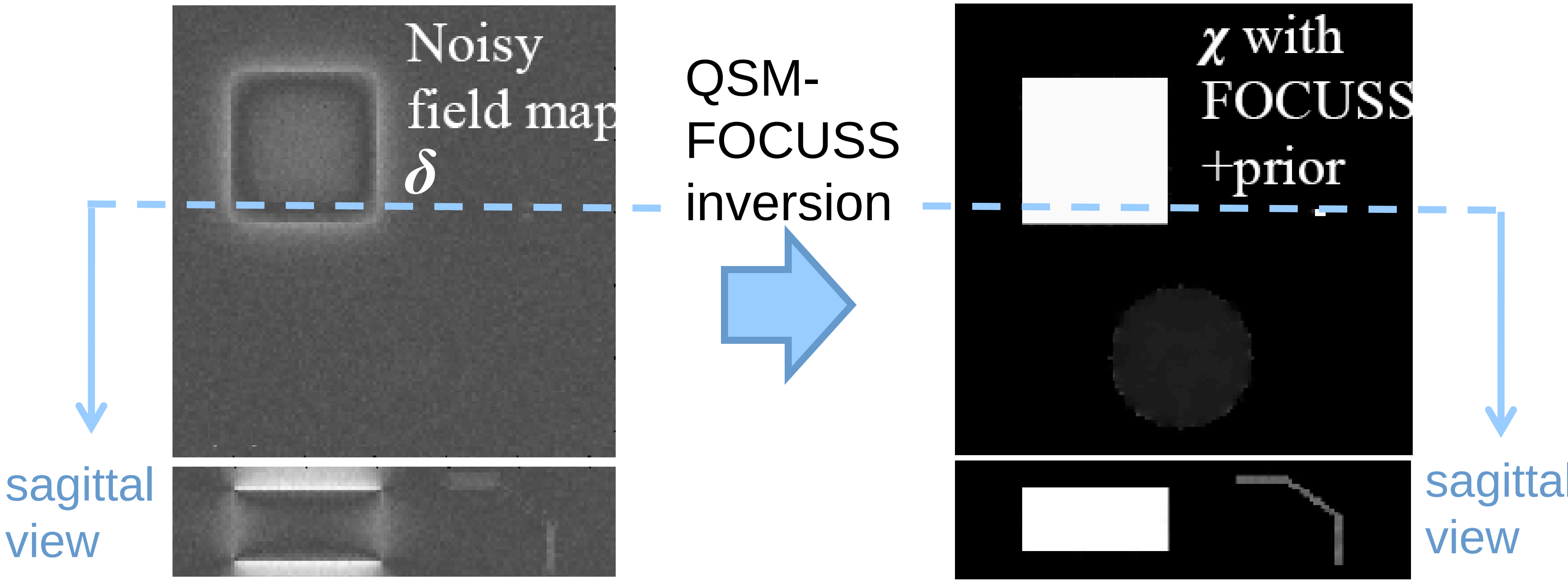
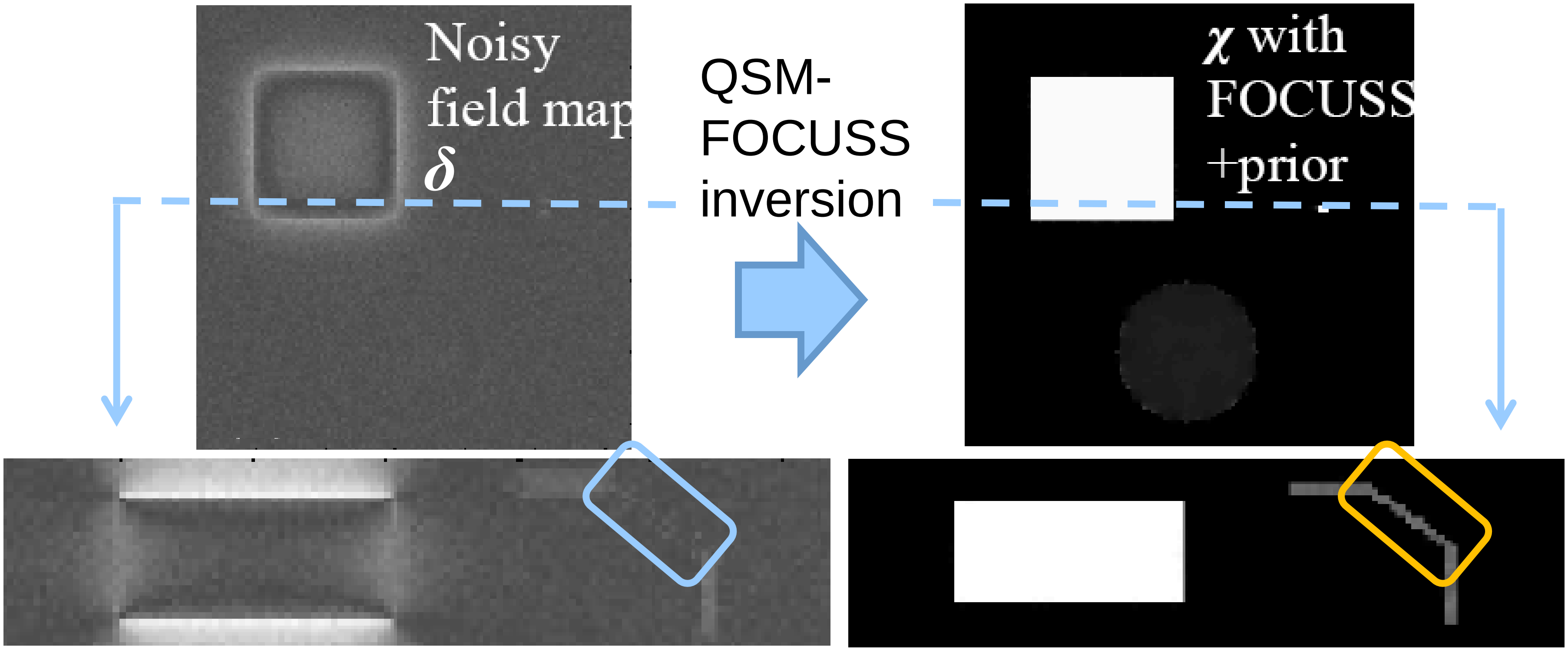
QSM result: FOCUSS-QSM with magnitude prior



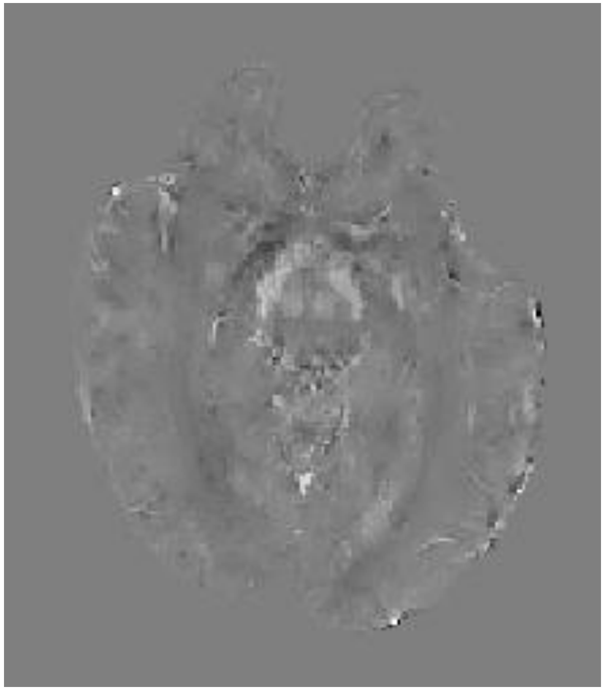
- Starting from the noisy field map δ, FOCUSS-QSM with magnitude prior yielded a susceptibility map with 1.3 % RMSE relative to true χ.

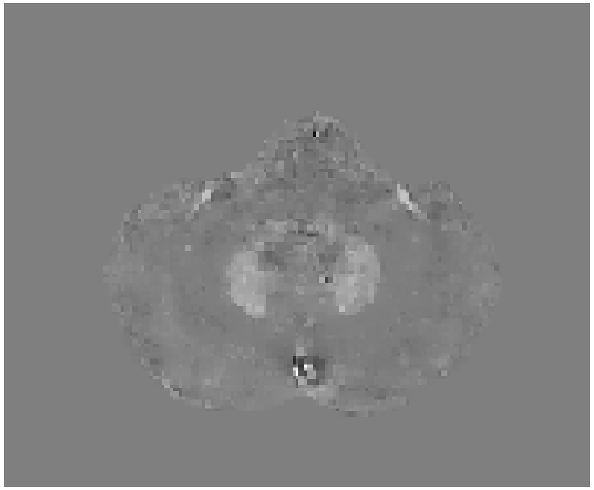
QSM result: FOCUSS-QSM with magnitude prior



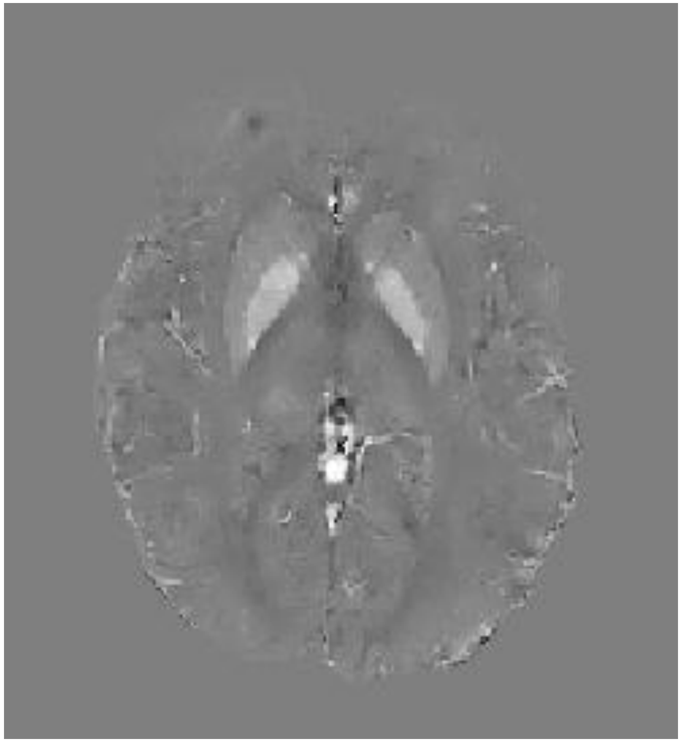
- The reconstructed susceptibility map managed to recover the vessel at the magic angle, which was virtually lost in the field map.

In vivo QSM result: FOCUSS-QSM with magnitude prior



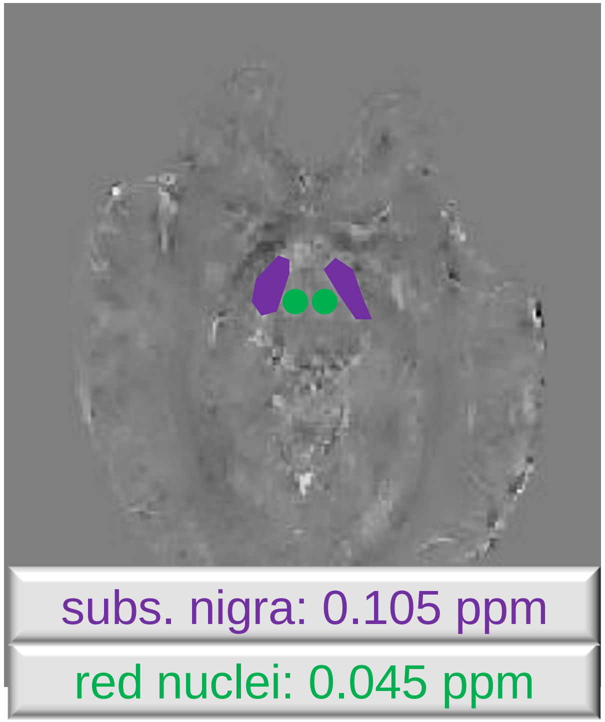


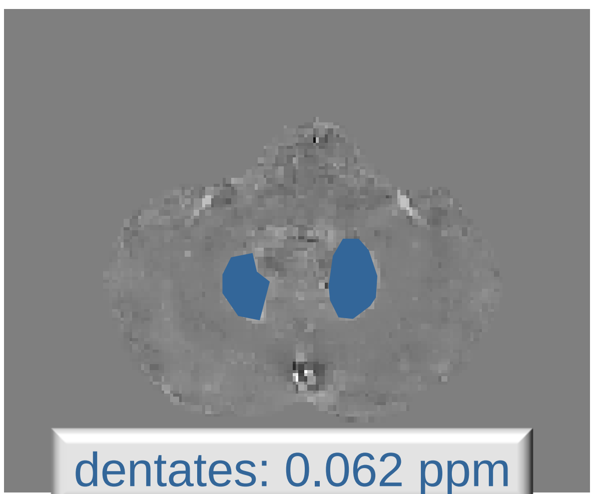
- 3D GRE acquisition at 3T
- 32 channel receive array
- 0.94x0.94x2.5 mm^3 resolution
- TE: 20 ms



-0.3 ppm  0.3 ppm

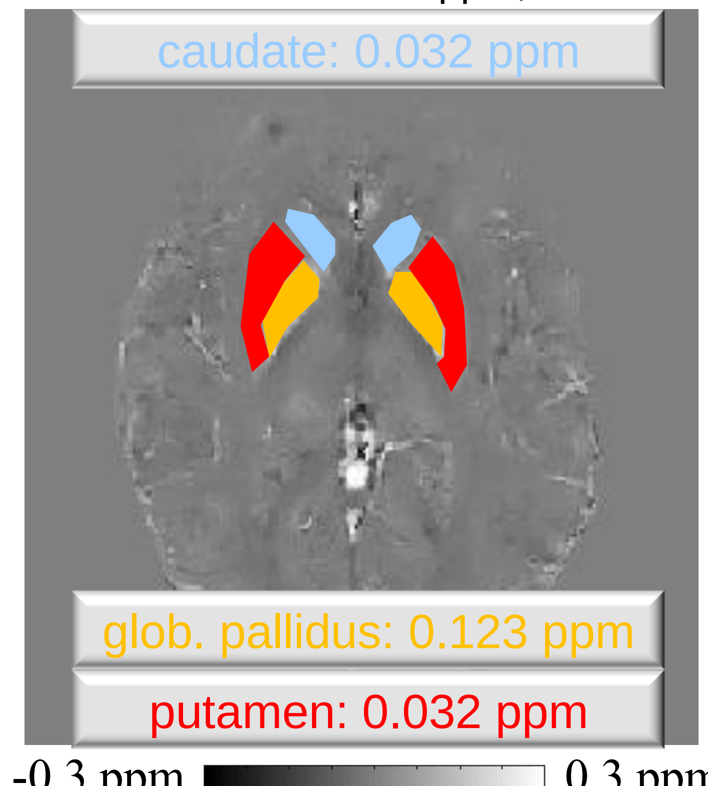
In vivo QSM result: FOCUSS-QSM with magnitude prior



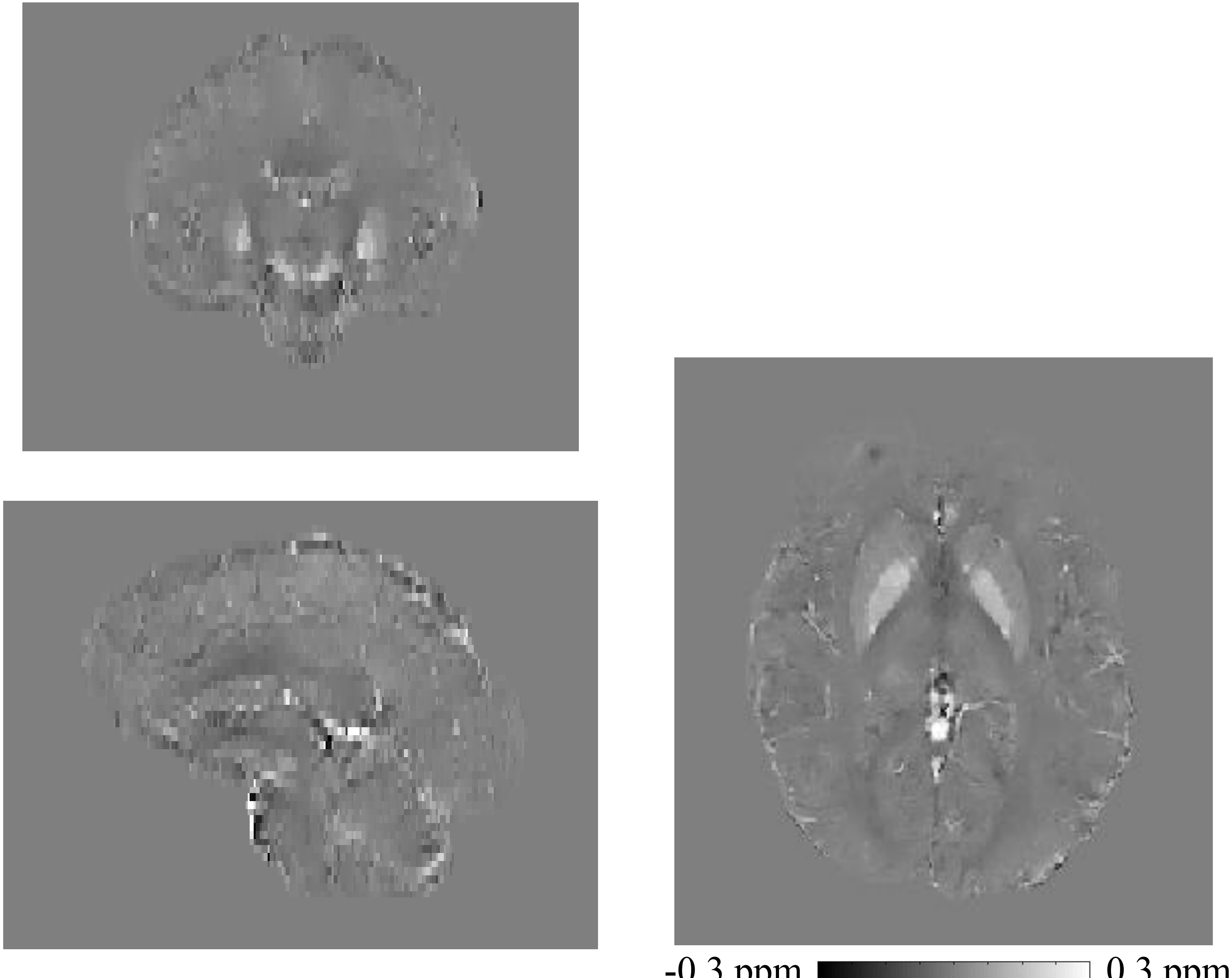


Structure	$\Delta\chi$ [ppm]
Globus Pallidus	12.3
Substantia Nigra	10.5
Dentate	6.2
Red Nucleus	4.5
Putamen	3.2
Caudate	2.3

x 0.01 ppm, relative to χ_{CSF}



In vivo QSM result: FOCUSS-QSM with magnitude prior



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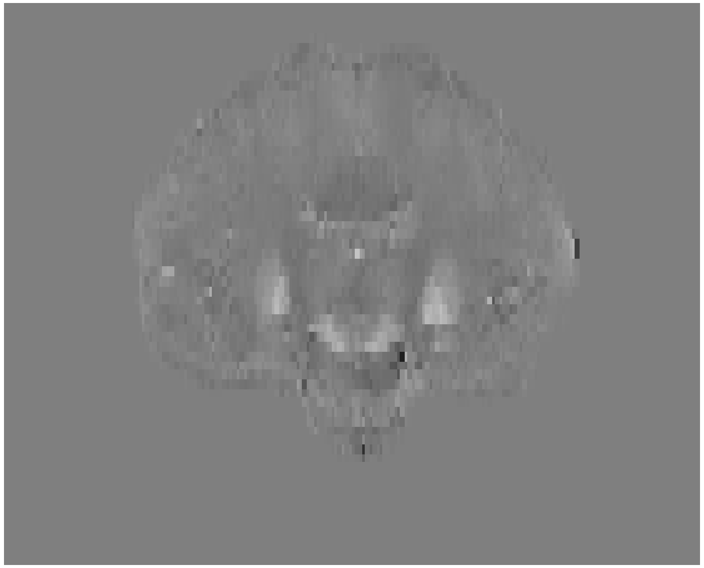
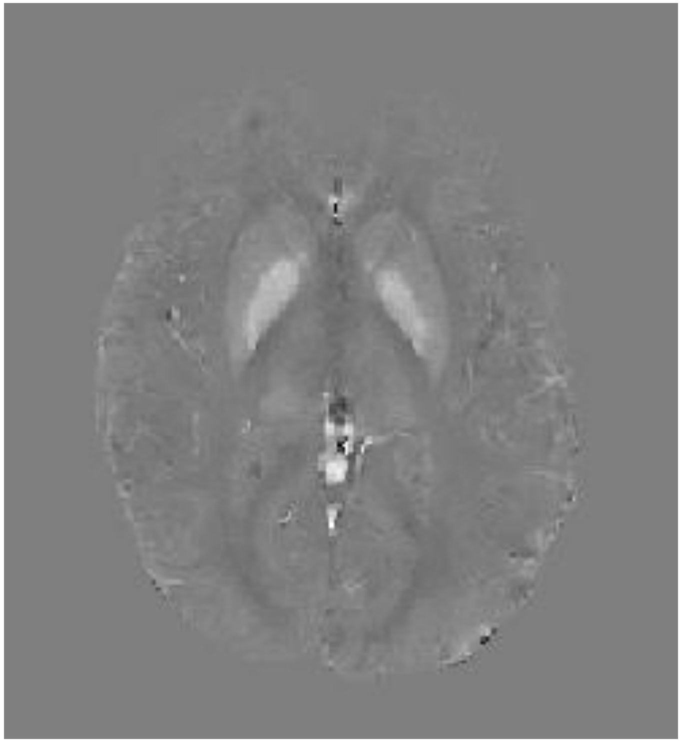
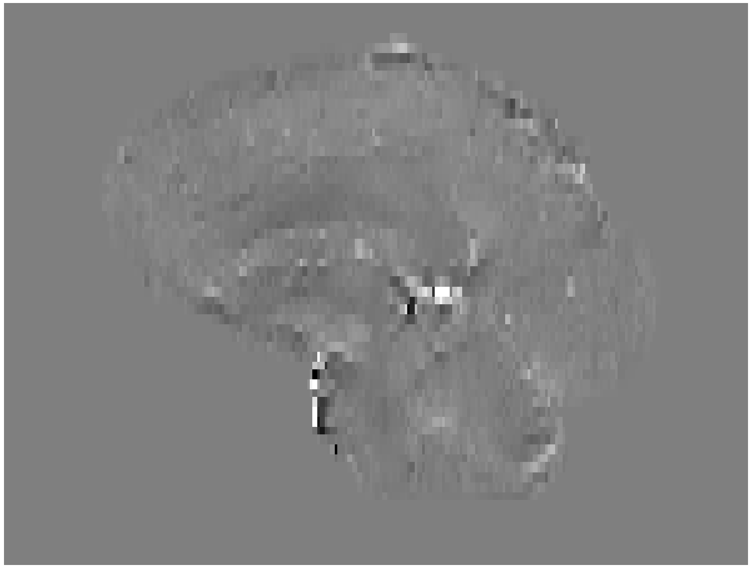
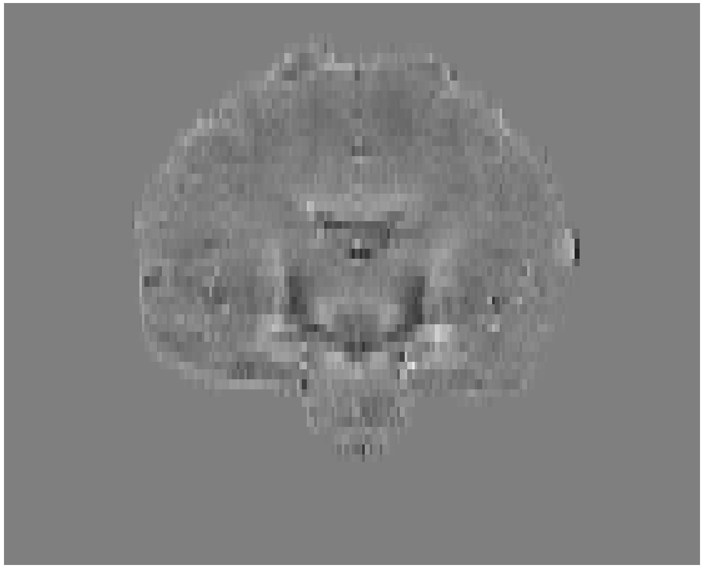


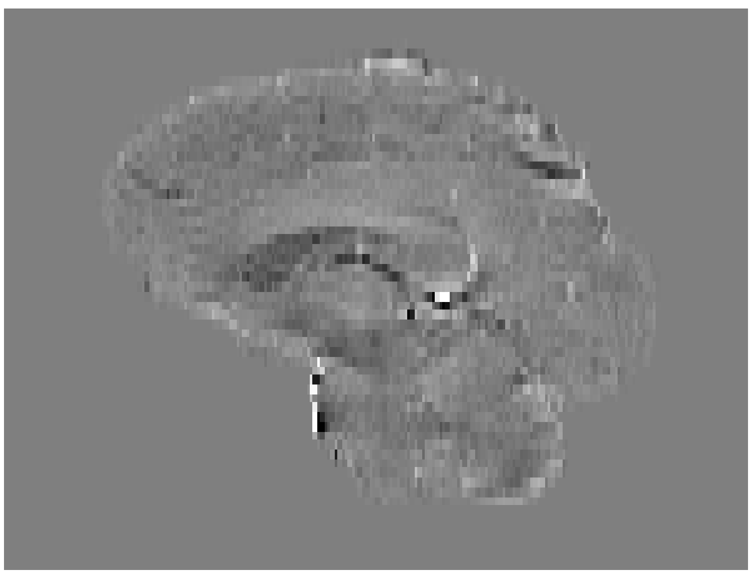
Vessels are less apparent without the magnitude prior

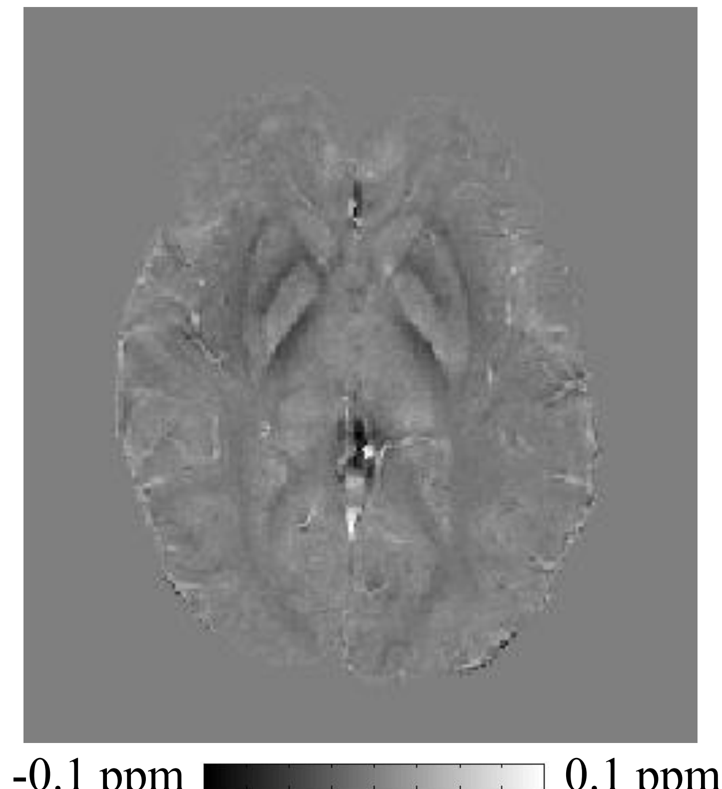




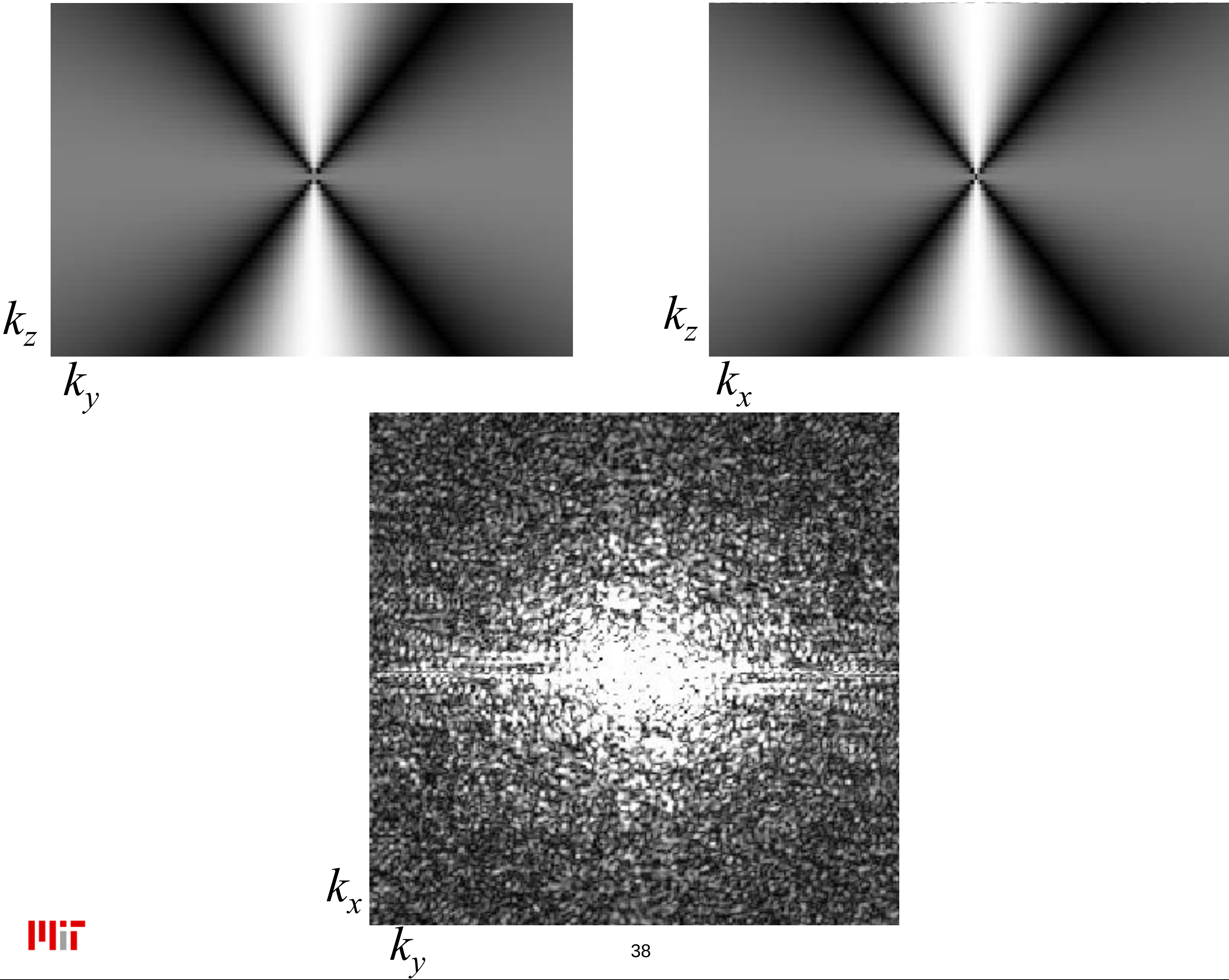
Corresponding Tissue Field Map:



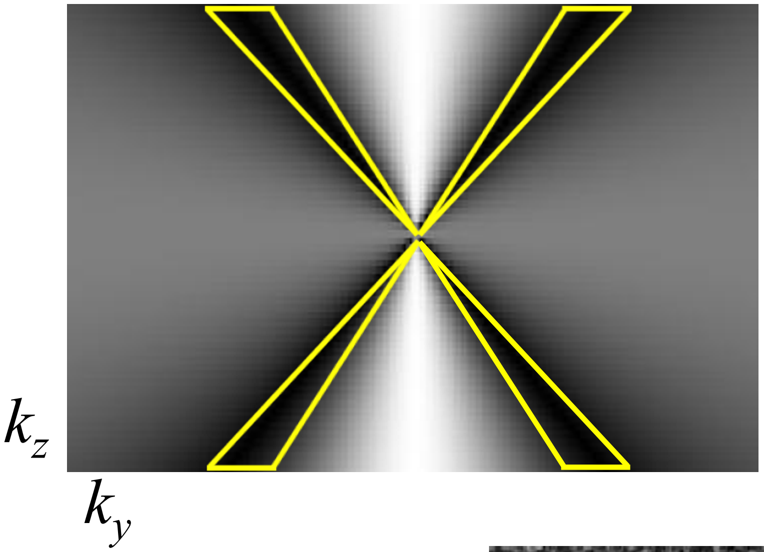


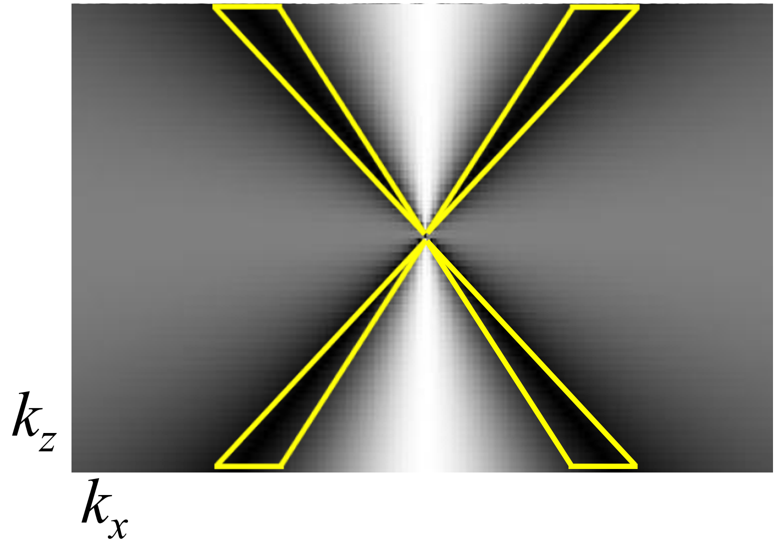


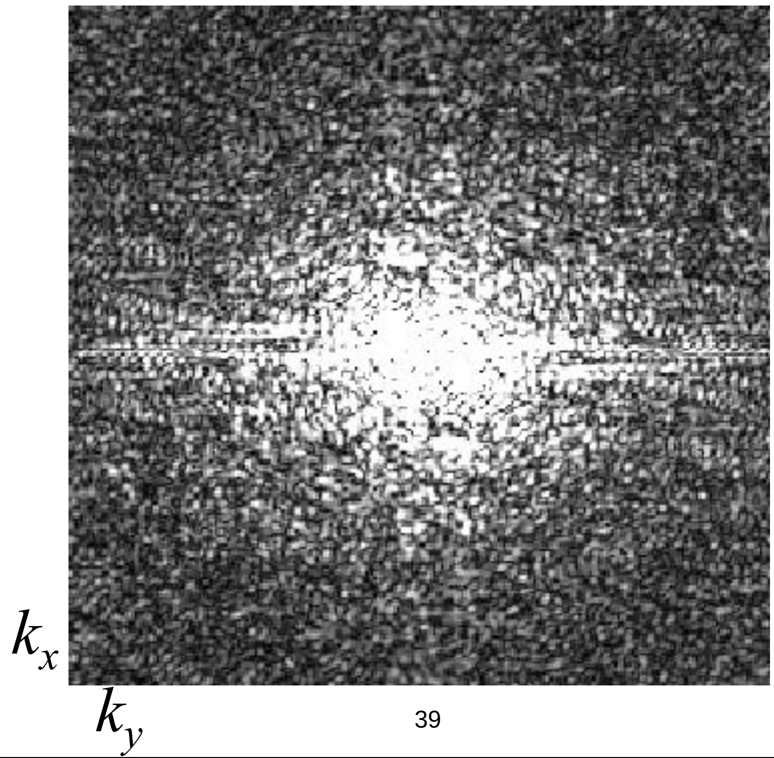
In vivo QSM result with magnitude prior in k-space:



In vivo QSM result with magnitude prior in k-space:







Potential drawbacks of FOCUSS-QSM

- Computation time:
  - Dipole fitting for background removal ≈ 2 hours
  - FOCUSS-QSM ≈ 1 hours
  - Total processing time ≈ **3 hours** for data of size [256x256x64]

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- Solution:
  - Both algorithms solve Least Squares problems, Graphics Processing Card (GPU) implementation will greatly enhance the performance

Conclusion

- Starting with a multi-coil 3D GRE acquisition, we outlined coil combination and background phase elimination methods that yielded the tissue field map.

- We introduced a Quantitative Susceptibility Mapping algorithm that makes use of the magnitude image to facilitate the kernel inversion.