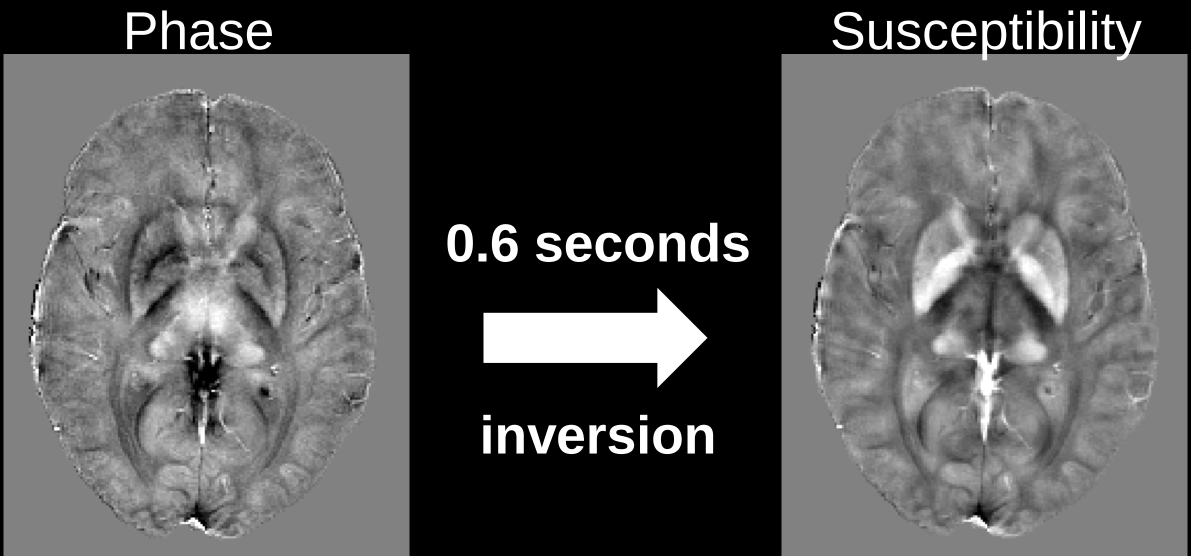

Phase

0.6 seconds

inversion

Susceptibility

Regularized QSM in Seconds

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"Discovery, Innovation & Application – Advancing MR for Improved Health"

Declaration of Relevant Financial Interests or Relationships

Speaker Name: Berkin Bilgic

I have no relevant financial interest or relationship to disclose with regard to the subject matter of this presentation.

Quantitative Susceptibility Mapping (QSM)

- Quantitative Susceptibility Mapping (QSM) aims to quantify tissue magnetic susceptibility χ
- Susceptibility correlates well with tissue iron concentration, especially in iron rich deep gray matter structures [1,2]

[1] Langkammer *et al*., Neuroimage 2012
[2] Bilgic *et al*., Neuroimage 2012

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- Susceptibility mapping requires the solution of an inverse problem,

$$\mathbf{F}^H \underset{\substack{\downarrow \\ \text{diagonal} \\ \text{matrix}}}{\mathbf{D}} \underset{\substack{\downarrow \\ \text{DFT}}}{\mathbf{F}} \; \underset{\substack{\downarrow \\ \text{unknown} \\ \text{susceptibility}}}{\chi} = \underset{\substack{\downarrow \\ \text{unwrapped} \\ \text{phase}}}{\phi}$$

[1] Langkammer *et al*., Neuroimage 2012
[2] Bilgic *et al*., Neuroimage 2012

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$$\mathbf{F}^{H}\ \mathbf{D}\ \mathbf{F}\ \underset{\substack{\downarrow \\ \text{to be estimated}}}{\boxed{\chi}} = \underset{\substack{\downarrow \\ \text{measured}}}{\boxed{\phi}}$$

[1] Langkammer *et al*., Neuroimage 2012
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Quantitative Susceptibility Mapping (QSM)

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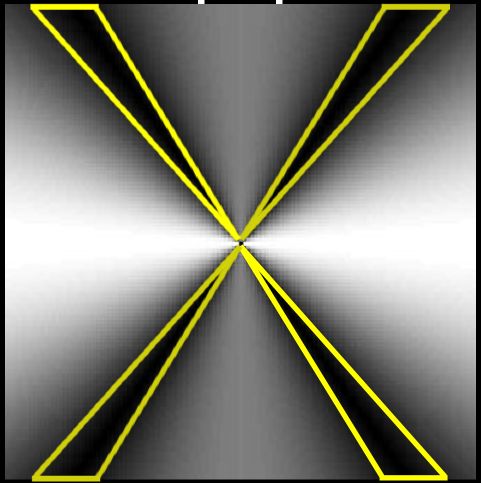
- Susceptibility mapping requires the solution of an inverse problem,

$$\mathbf{F}^H \, \mathbf{D} \, \mathbf{F} \; \chi = \phi$$

$$\mathbf{D} = \frac{1}{3} - \frac{{k_z}^2}{k^2}$$

⇨ Undersamples k-space on a conical surface



|D|

Regularized QSM

- Solution of inverse problem is facilitated by regularization that imposes prior knowledge [1]

$$\chi = argmin_{\chi}\underbrace{\|\boldsymbol{\phi} - \mathbf{F}^{H}\mathbf{D}\mathbf{F}\boldsymbol{\chi}\|_{2}{}^{2}}_{\text{data consistency}} + \lambda \cdot \underbrace{\|\mathbf{G}\boldsymbol{\chi}\|_{2}{}^{2}}_{\ell_2 \text{ over gradients}}$$

[1] de Rochefort *et al*., Magn Reson Med 2010

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$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{\boldsymbol{x}} \\ \mathbf{G}_{\boldsymbol{y}} \\ \mathbf{G}_{\boldsymbol{z}} \end{bmatrix}$$

gradient in 3D

[1] de Rochefort *et al*., Magn Reson Med 2010

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- Prior: underlying susceptibility map is smooth

[1] de Rochefort *et al.*, Magn Reson Med 2010

Regularized QSM

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- Existing methods work iteratively [1,2], requiring

  ~30 minutes for a 3D volume → not feasible

- We address this with fast recon in ~1 second

[1] de Rochefort *et al*., Magn Reson Med 2010
[2] Bilgic *et al*., NeuroImage 2012

Regularized QSM

- Solution of inverse problem is facilitated by regularization that imposes prior knowledge [1]

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- Solution can be evaluated in closed-form

$$\chi = (\mathbf{F}^{H}\mathbf{D}^{2}\mathbf{F} + \lambda \cdot \mathbf{G}^{H}\mathbf{G})^{-1}\mathbf{F}^{H}\mathbf{D}\mathbf{F}\boldsymbol{\phi}$$

- The minimizer can be computed efficiently given that the matrix inversion is rapidly performed

[1] de Rochefort *et al.*, Magn Reson Med 2010

Fast Regularized QSM

- Solution can be evaluated in closed-form

$$\chi = (\mathbf{F}^H \mathbf{D}^2 \mathbf{F} + \lambda \cdot \mathbf{G}^H \mathbf{G})^{-1} \mathbf{F}^H \mathbf{D} \mathbf{F} \boldsymbol{\phi}$$

Fast Regularized QSM

- Solution can be evaluated in closed-form

$$\chi = (\mathbf{F}^H \mathbf{D}^2 \mathbf{F} + \lambda \cdot \mathbf{G}^H \mathbf{G})^{-1} \mathbf{F}^H \mathbf{D} \mathbf{F} \boldsymbol{\phi}$$

- Gradient along x-axis can be represented in k-space by multiplication with a diagonal matrix $\mathbf{E}_{\boldsymbol{x}}$

$$\mathbf{G}_{\boldsymbol{x}} = \mathbf{F}^H \mathbf{E}_{\boldsymbol{x}} \mathbf{F}$$

where $\mathbf{E}_x(i,i) = 1 - e^{(-2\pi\sqrt{-1}k_x(i,i)/N_x)}$

Fast Regularized QSM

- Solution can be evaluated in closed-form

$$\chi = (\mathbf{F}^H \mathbf{D}^2 \mathbf{F} + \lambda \cdot \mathbf{G}^H \mathbf{G})^{-1} \mathbf{F}^H \mathbf{D} \mathbf{F} \boldsymbol{\phi}$$

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- $\mathbf{E}_{\boldsymbol{x}}$ is simply the k-space representation of the difference operator $\delta_x - \delta_{x-1}$

Fast Regularized QSM

- Solution can be evaluated in closed-form

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- With this formulation, closed-form solution becomes

$$\chi = \mathbf{F}^H\mathbf{D}\,\underbrace{\left[\mathbf{D}^2 + \lambda\cdot(\mathbf{E}_x^2 + \mathbf{E}_y^2 + \mathbf{E}_z^2)\right]}_{\text{all matrices diagonal}}{}^{-1}\,\mathbf{F}\boldsymbol{\phi}$$

Fast Regularized QSM

- Solution can be evaluated in closed-form

$$\boldsymbol{\chi} = (\mathbf{F}^H \mathbf{D}^2 \mathbf{F} + \lambda \cdot \mathbf{G}^H \mathbf{G})^{-1} \mathbf{F}^H \mathbf{D} \mathbf{F} \boldsymbol{\phi}$$

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- **Total cost:** Two FFTs and multiplication of diagonal matrices

Contributions

- Proposed closed-form method is 1000-times faster than iterative Conjugate Gradient solver in [1,2]

[1] de Rochefort *et al*., MRM 2010
[2] Bilgic *et al*., Neuroimage 2012

Contributions

- Proposed closed-form method is 1000-times faster than iterative Conjugate Gradient solver in [1,2]
- Proposed method yields exact minimizer while iterative methods converge to it

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Contributions

- Proposed closed-form method is 1000-times faster than iterative Conjugate Gradient solver in [1,2]
- Proposed method yields exact minimizer while iterative methods converge to it
- Automatic selection of regularization parameter λ is possible:

  Trace L-curve with closed-form method in a minute

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Contributions

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- Proposed method yields exact minimizer while iterative methods converge to it
- Automatic selection of regularization parameter λ is possible:

  Trace L-curve with closed-form method in a minute
- Combined with fast background removal methods like SHARP [3], enables real-time QSM

[1] de Rochefort *et al.*, MRM 2010
[2] Bilgic *et al.*, Neuroimage 2012
[3] Schweser *et al.*, MRM 2012

Comparison of methods

- **Proposed method:**
  - ❖Closed form QSM
- **Previous method:**
  - ❖Iterative QSM with Conjugate Gradient [1,2]

    converges to closed-form solution

[1] de Rochefort *et al*., MRM 2010
[2] Bilgic *et al*., Neuroimage 2012
[3] Shmueli *et al*., MRM 2009

Comparison of methods

- **Proposed method:**
  - ❖ Closed form QSM

- **Previous method:**
  - ❖ Iterative QSM with Conjugate Gradient [1,2]
    converges to closed-form solution
  - ❖ Initialize with Thresholded K-space Division map [3]
  - ❖ Terminate when change in susceptibility is less than 1%

[1] de Rochefort *et al.*, MRM 2010
[2] Bilgic *et al.*, Neuroimage 2012
[3] Shmueli *et al.*, MRM 2009

Regularized QSM Methods

- Numerical Phantom
  - ☐ Three compartments (gray, white, CSF) with constant χ
  - ☐ Phase ϕ computed from true χ, and Gaussian noise added
  - ☐ Regularization param λ chosen to minimize RMSE in χ recon

[1] Liu *et al*., NMR in Biomed 2011

Regularized QSM Methods

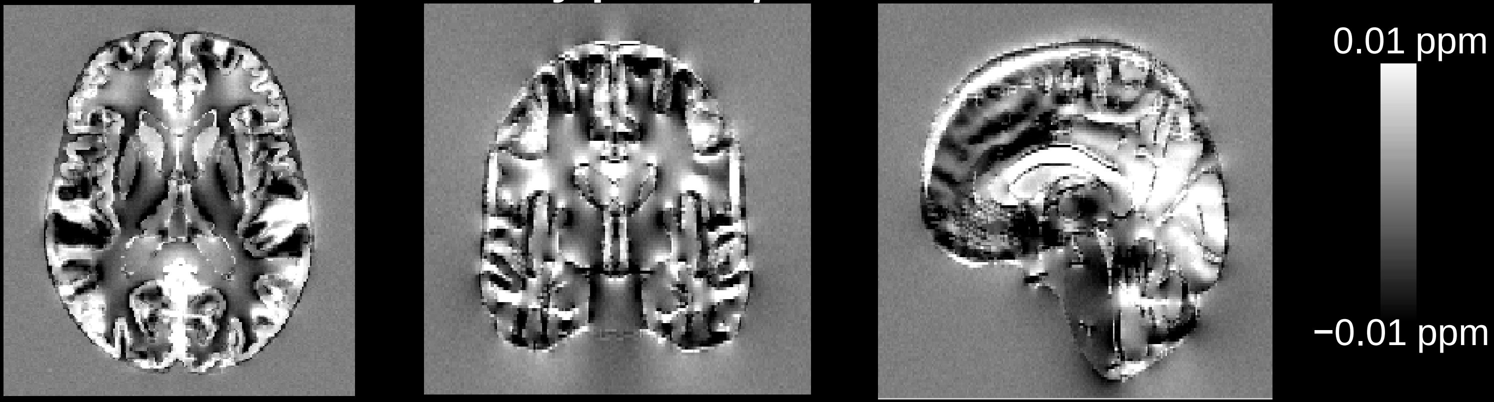
- Numerical Phantom
  - ☐ Three compartments (gray, white, CSF) with constant χ
  - ☐ Phase ϕ computed from true χ, and Gaussian noise added
  - ☐ Regularization param λ chosen to minimize RMSE in χ recon
- In Vivo 3D SPGR
  - ☐ Healthy subject at 1.5T with resolution 0.94×0.94×2.5mm^3
  - ☐ Regularization parameter λ chosen based on L-curve
  - ☐ Background phase removal with dipole fitting [1]
- Computations done on workstation with 32 CPU processors and 128 GB memory

[1] Liu *et al.*, NMR in Biomed 2011

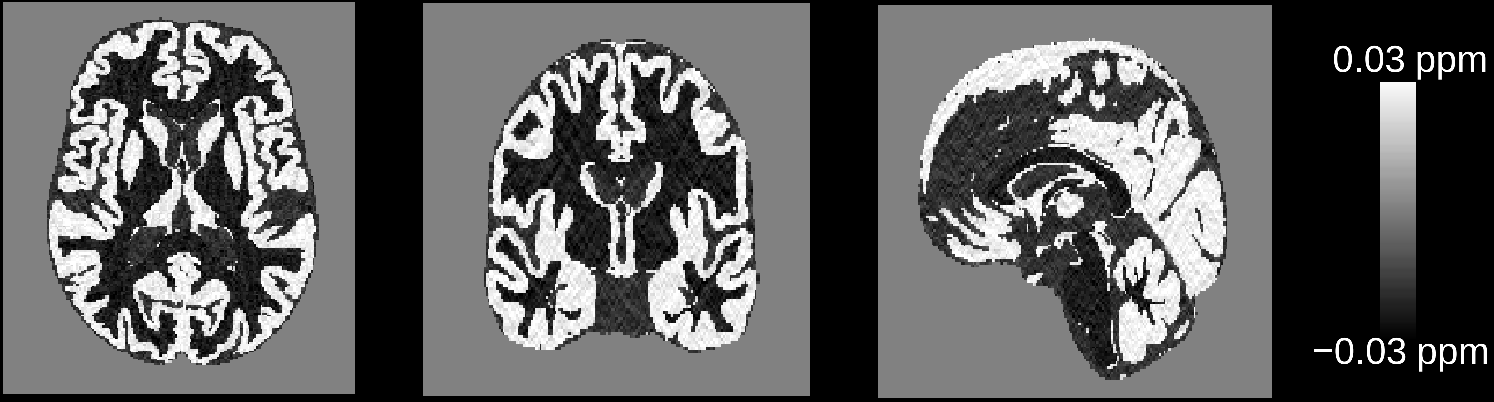
Numerical Phantom

error due to noise:
5.0% RMSE

Noisy phase ϕ

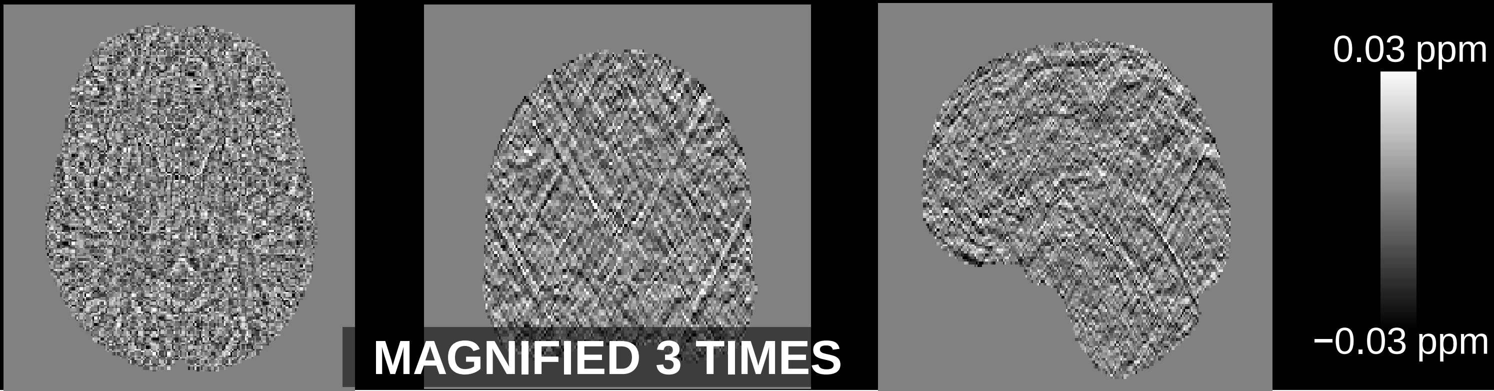


Closed-form QSM in 1.1 seconds



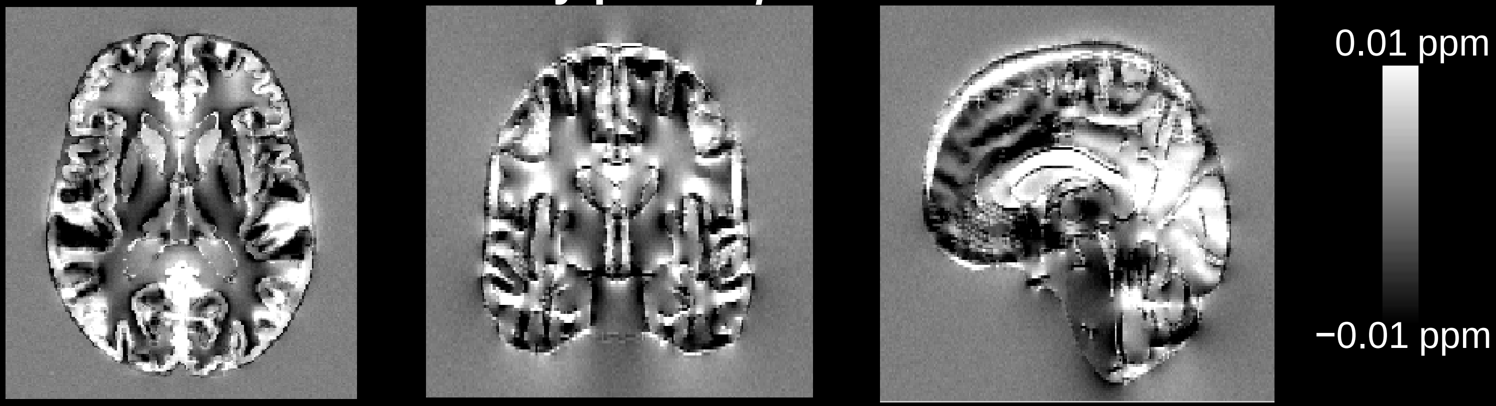
True χ known

Closed-form QSM error relative to True χ



MAGNIFIED 3 TIMES

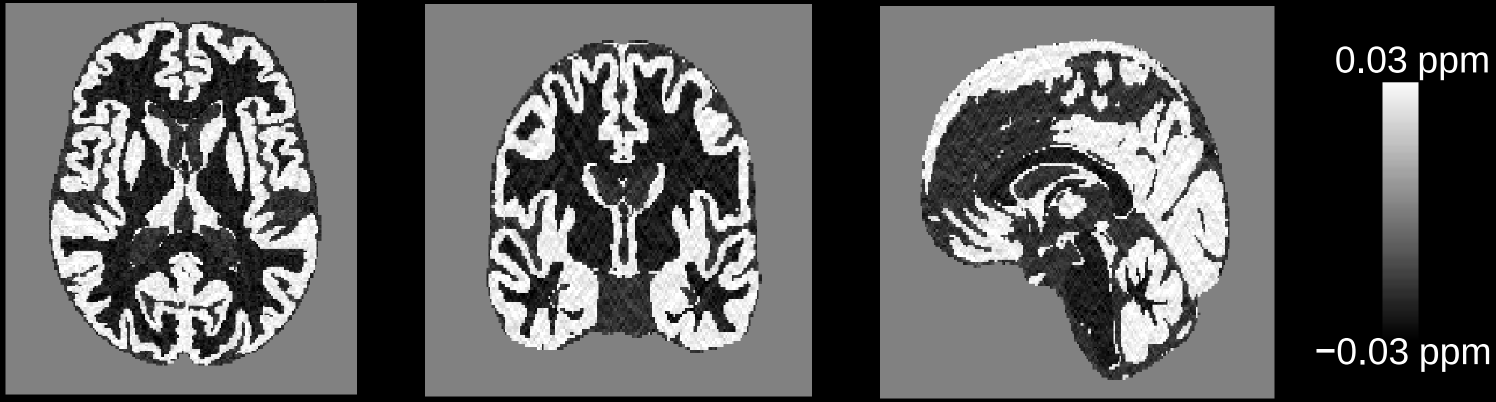
Numerical Phantom



Noisy phase ϕ

error due to noise:
5.0% RMSE

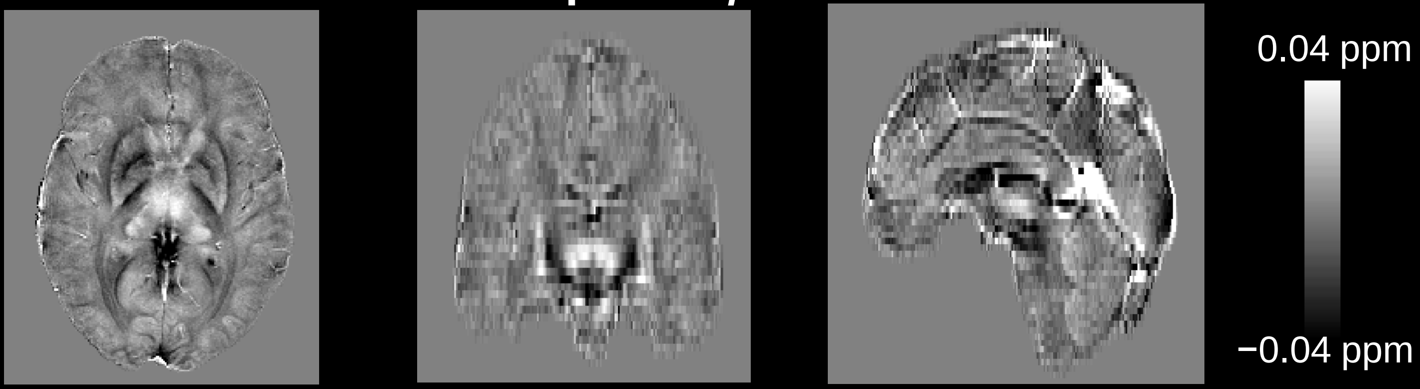
Closed-form QSM in 1.1 seconds



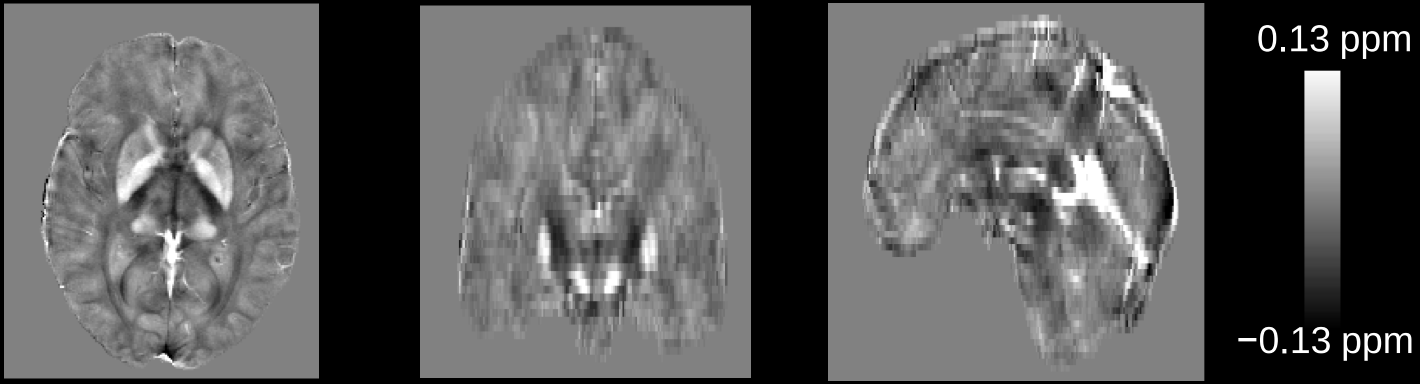
QSM Method	**Recon Time**	**Error relative to True χ**
Proposed Closed-Form	**1.1 seconds**	16.1 % RMSE
Conjugate Grad, 80 iters	33 minutes	16.8 % RMSE

In Vivo QSM

Tissue phase ϕ

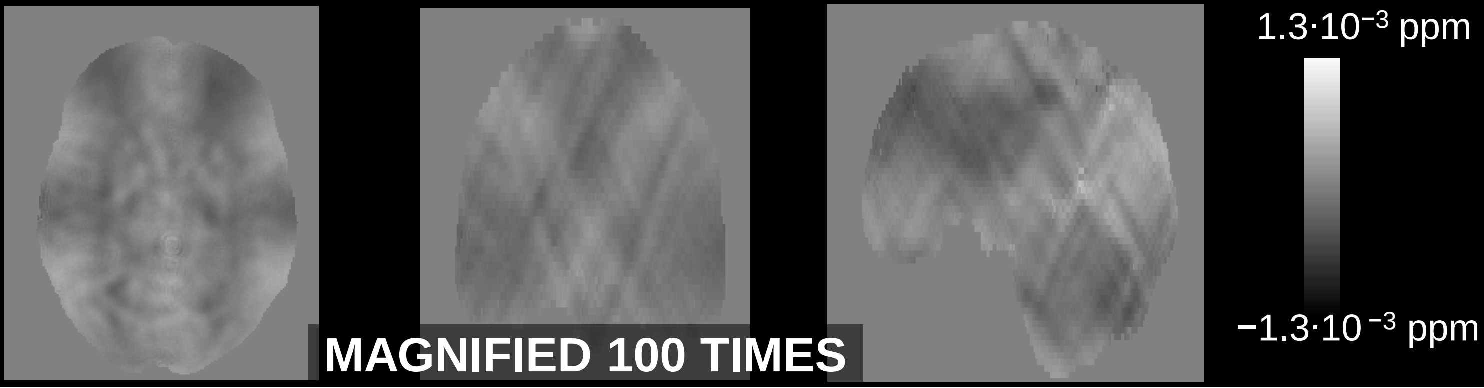


Closed-form QSM in 0.6 seconds



True χ
not known

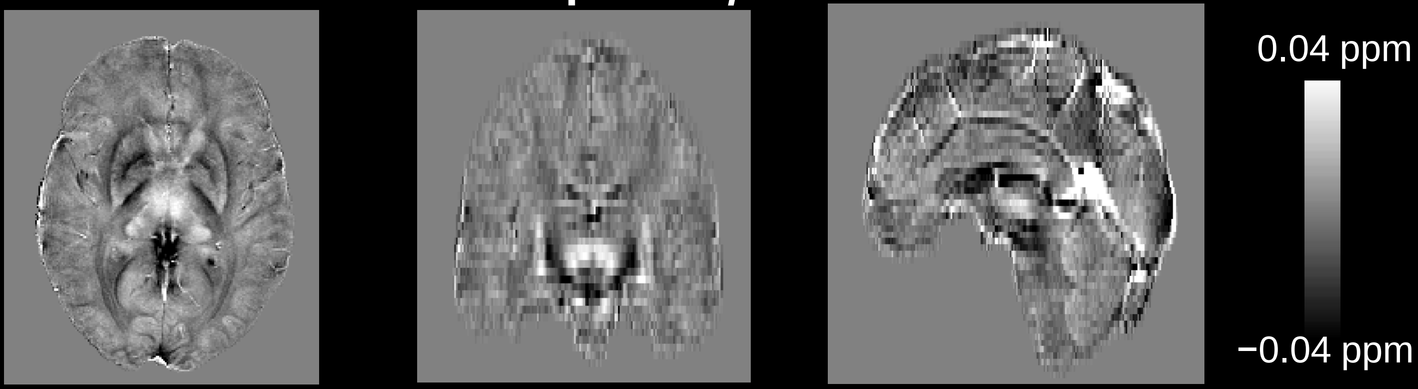
Closed-form and Iterative QSM difference: 0.6%



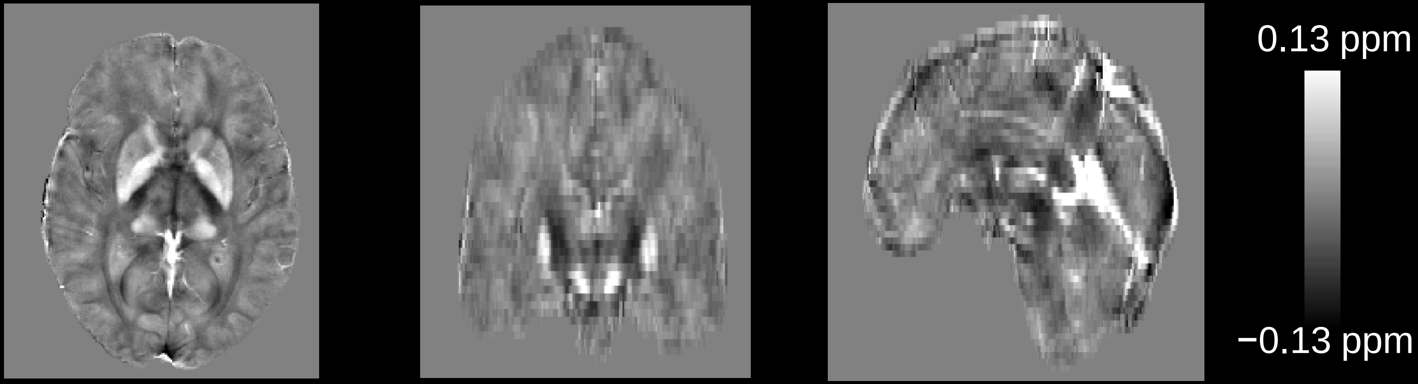
MAGNIFIED 100 TIMES

In Vivo QSM

Tissue phase ϕ



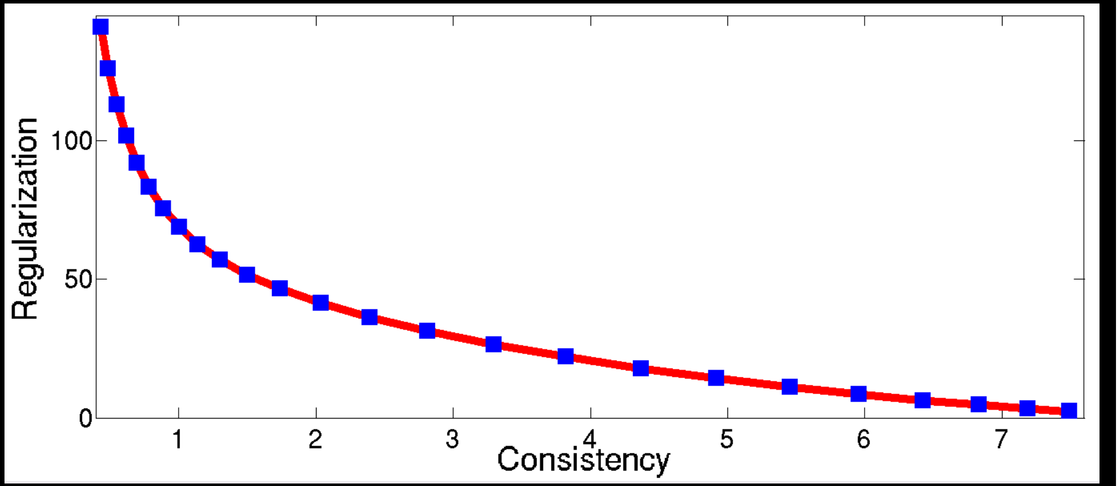
Closed-form QSM in 0.6 seconds



QSM Method	Recon Time
Proposed Closed-Form	**0.6 seconds**
Conjugate Gradient, 80 iters	18 minutes

Tracing the L-curve

$\|\mathbf{G}\chi\|_2$

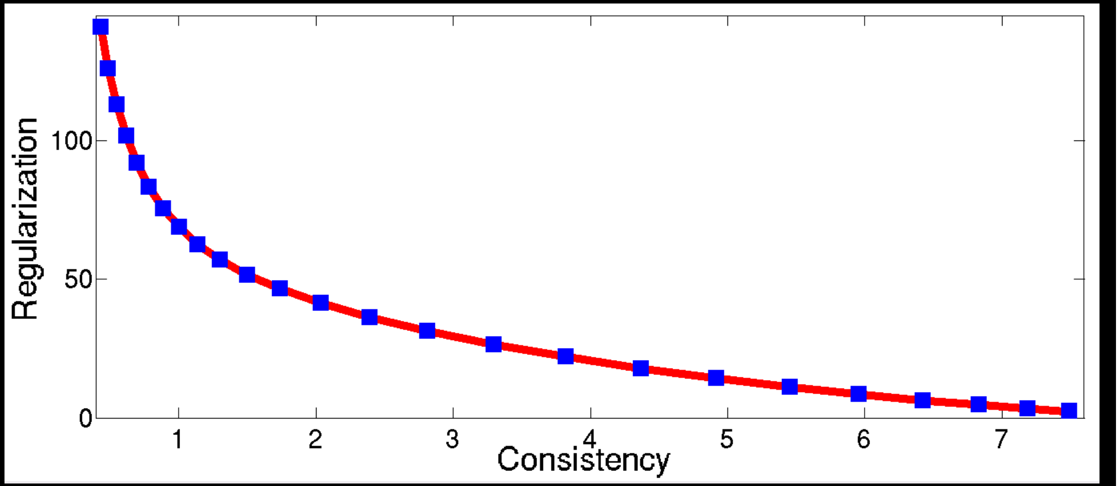


$\|\boldsymbol{\phi} - \mathbf{F}^H\mathbf{D}\mathbf{F}\chi\|_2$

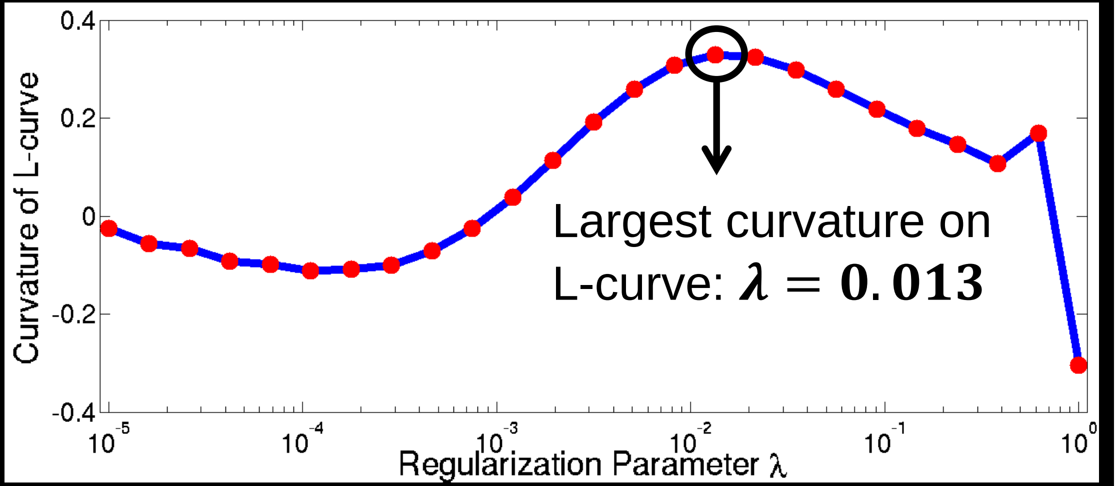
- Computing χ for 25 different values of λ: 50 seconds

Tracing the L-curve

$\|\mathbf{G}\chi\|_2$



$\|\phi - \mathbf{F}^H\mathbf{D}\mathbf{F}\chi\|_2$

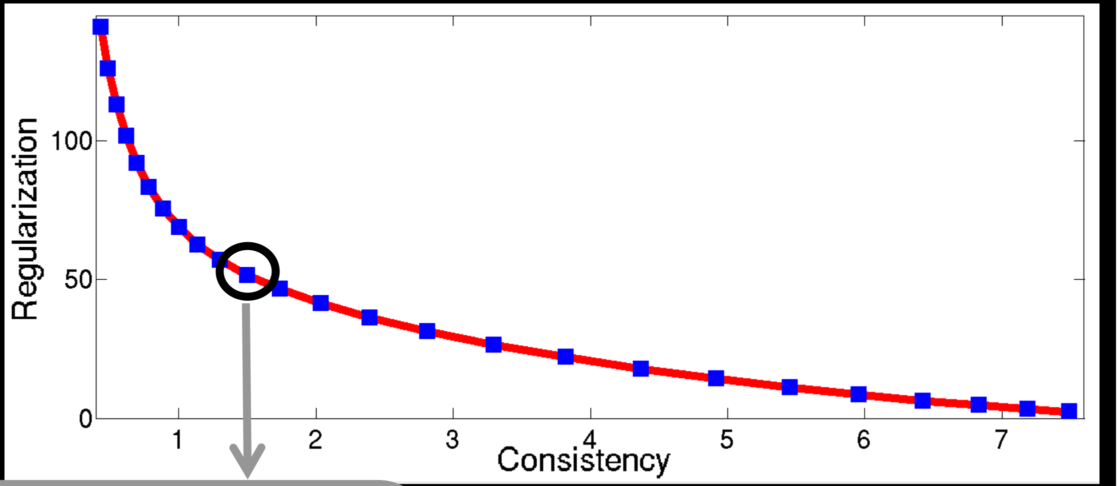


Largest curvature on L-curve: $\boldsymbol{\lambda} = \mathbf{0.013}$

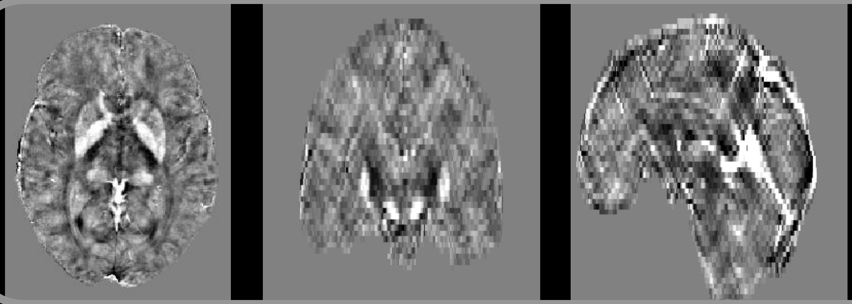
- Computing χ for 25 different values of λ: 50 seconds
- Find optimal λ by computing the curvature of L-curve

Tracing the L-curve

$\|\mathbf{G}\chi\|_2$



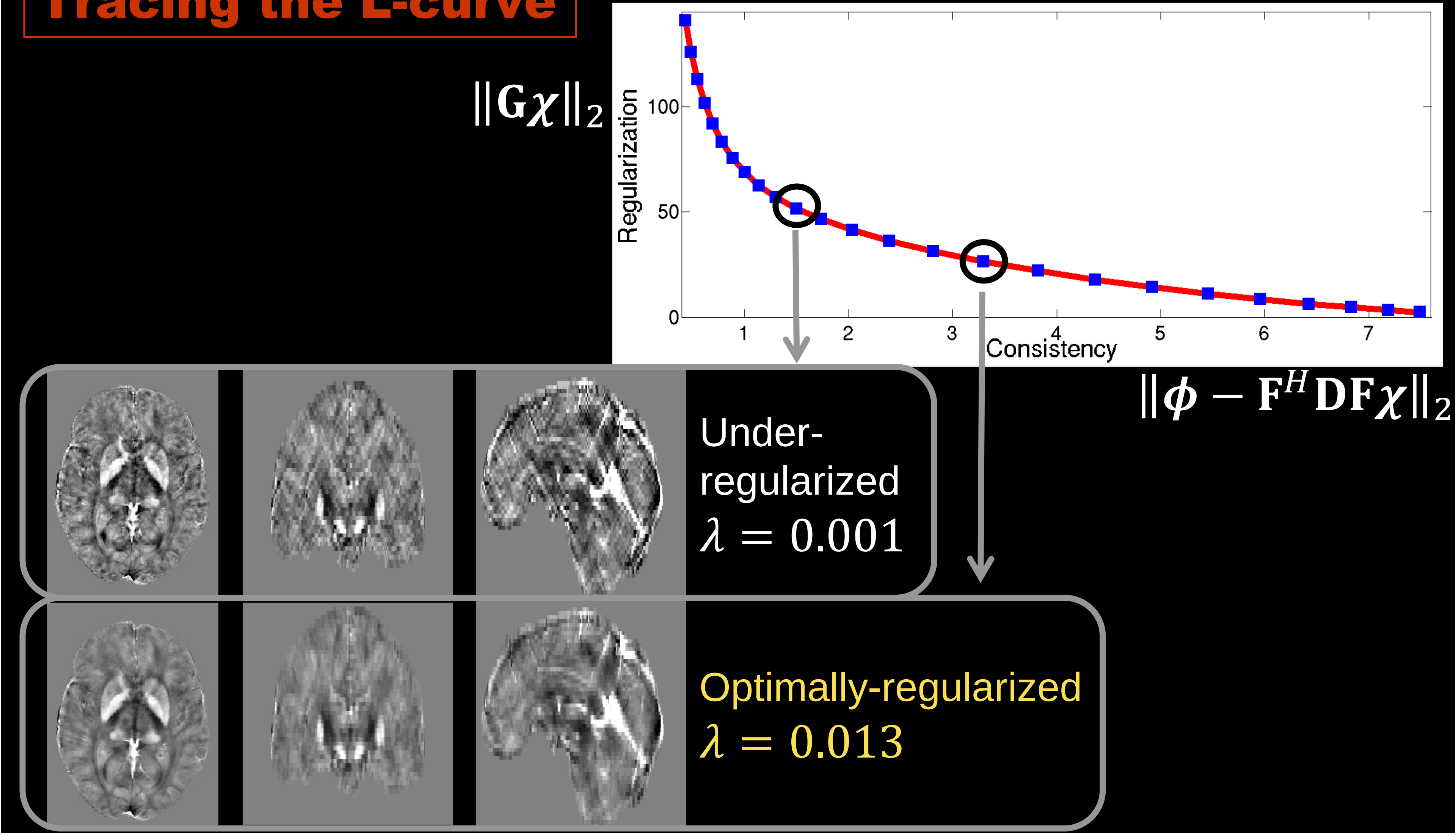
$\|\boldsymbol{\phi} - \mathbf{F}^H\mathbf{D}\mathbf{F}\chi\|_2$



Under-regularized

$\lambda = 0.001$

Tracing the L-curve



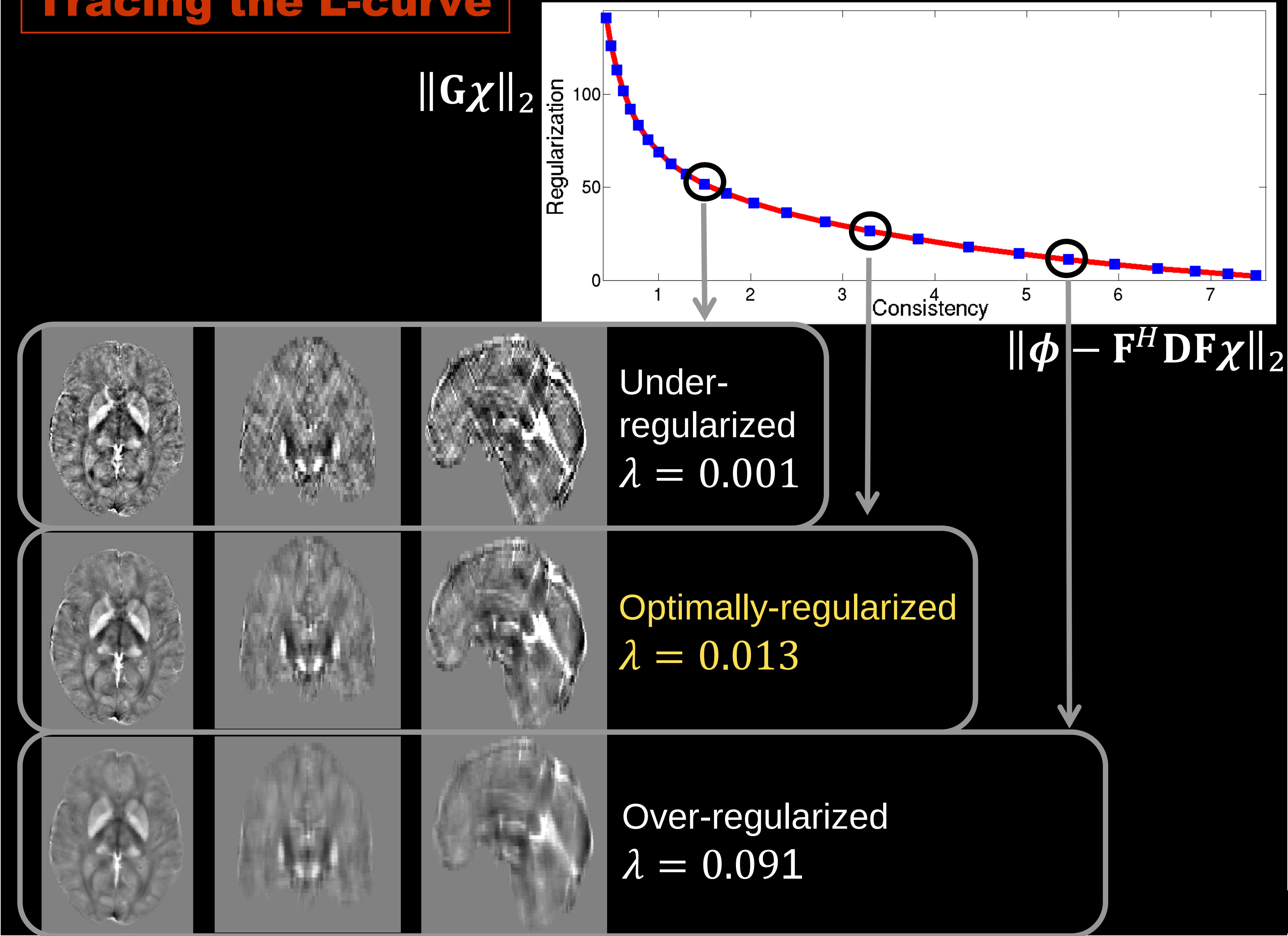
$\|\mathbf{G}\chi\|_2$

$\|\boldsymbol{\phi} - \mathbf{F}^H\mathbf{D}\mathbf{F}\chi\|_2$

Under-regularized
$\lambda = 0.001$

Optimally-regularized
$\lambda = 0.013$

Tracing the L-curve



$\|\mathbf{G}\chi\|_2$

$\|\phi - \mathbf{F}^H\mathbf{D}\mathbf{F}\chi\|_2$

Under-regularized
$\lambda = 0.001$

Optimally-regularized
$\lambda = 0.013$

Over-regularized
$\lambda = 0.091$

Conclusion

- Proposed closed form recon for L2-regularized QSM
- 1000-times faster recon compared to Conjugate Gradient solver [1,2]
- Automatic selection for λ feasible with L-curve in a minute
- Software Download:

http://web.mit.edu/berkin/www/software.html

[1] de Rochefort *et al*., MRM 2010
[2] Bilgic *et al*., Neuroimage 2012

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